

A Study of the Transfer of Heat in the Motion of a Second-Order Fluid Through a Channel with Porous Walls While under the Influence of a Transversal Magnetic Field

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Abstract:

The purpose of this study is to examine heat transfer in the flow of a second-order fluid through a porous-walled channel subject to a transverse magnetic field using a regular perturbation approach. The second order impacts on the temperature profile are demonstrated for a range of Hartman and Reynolds numbers. The results can also be achieved for Newtonian fluids by resetting the second-order parameter to zero.

INTRODUCTION

Heat transfer in the flow of an electrically conducting fluid through porous barriers is of practical importance. This applies to concerns pertaining to gaseous diffusion as well as other applications. Terrill and Shrestha investigated the effects of a magnetic field on the fluid's electrical conductivity as well as the problem of constant laminar flow of an incompressible viscous fluid in a two-dimensional channel with walls of varying permeability. In addition, they looked at the effects of the problem on the electrical conductivity of the fluid. Agrawal investigated the issue of second-order fluid flow while simultaneously accounting for heat transfer in a conduit with porous walls. Sharma and Singh conducted research on the computational solution of a second-order fluid flow via a porous channel when a transverse magnetic field was present. The purpose of this study is to examine the process of heat transfer that occurs during the flow of a second-order fluid through a channel that has porous walls while a transverse magnetic field is present. The approach that will be used is called a regular perturbation approach. The second-order impacts on the temperature profile are demonstrated here for a range of different Hartman and Reynolds numbers. The conclusions can also be achieved for Newtonian fluids by putting a value of zero in place of the second-order parameter.

THE PROBLEM'S FORMULATION

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The process of studying the heat transfer in a two-dimensional steady flow of an incompressible second-order fluid in a channel with a width of $2h$ and two porous walls of equal permeability (coinciding with the plane $y = h$) is called a two-dimensional steady flow of an incompressible second-order fluid. The entire system of the channel is built in such a way that both the bottom and the top are totally insulated and do not conduct heat in any direction. A continuous magnetic field that is applied in a normal direction to the channel axis is denoted by " H_0 ." Because the magnetic Reynolds number is quite low, the induced magnetic field in the flow has been ignored. The two walls of the channel are subjected to a constant suction in the shape of a V. Let's choose an x -axis and a y -axis on a plane that are, respectively, parallel and perpendicular to the walls of the channel. Let's say that u and v stand for the directional components of the velocity in the x and y directions, respectively.

In order to keep track of Terrill and Shrestha, a stream function is being employed.

$$\psi(x, y) = (hU - Vx) f(\eta) \quad (1.1)$$

Where U denotes the entry velocity, $(\eta = y/h)$ denotes the dimensionless distance, and $2h$ is the distance between the channel walls. Terril and Shrestha's velocity field in non-dimensional form is as follows:

$$u = U \left(\frac{d\psi}{dx} \right) \quad v = -V \left(\frac{d\psi}{dy} \right) \quad (1.2)$$

Where the dash represents a distinction with regard to. According to the formula (1.2), u is a function of x and v is a function of η alone. The constitutive equation (1.4), as well as the equations of continuity and momentum, may be expressed as follows:

$$u/x + (1/h)(v/\eta) = 0 \quad (1.3)$$

$$\begin{aligned} & \left(\frac{v}{h} \right) \frac{d}{d\eta} \left(\frac{u}{x} \right) + \left(\frac{v}{h} \right) \frac{d}{d\eta} \left(\frac{v}{\eta} \right) = \left(\frac{p}{x} \right) + \left(\frac{v}{h^2} \right) - \left(\frac{1}{p} \right) \left(\frac{p}{x} \right) + \left(\frac{v}{h^2} \right) \left[\left(\frac{2u}{\eta} \right) + v^2 \left[\left(\frac{1}{h^2} \right) \right] \right] \\ & \left[\left(\frac{2u}{\eta} \right) + v^2 \left[\left(\frac{2}{\eta} \right) \frac{u}{x} + \left(\frac{v}{h} \right) \left(\frac{v}{\eta} \right) + \left(\frac{2}{h^2} \right) \left(\frac{1}{\eta} \right) + \left(\frac{v}{h} \right) \left(\frac{v}{\eta} \right) + \left(\frac{2}{h^2} \right) \left(\frac{1}{\eta} \right) \right] \right] \\ & \left[\left(\frac{u}{x} \right) \left(\frac{v}{\eta} \right) \right] \left[\left(\frac{u}{x} \right) \left(\frac{v}{\eta} \right) \right] \left[\left(\frac{u}{x} \right) \left(\frac{v}{\eta} \right) + \left(\frac{v^3}{h^2} \right) \left(\frac{\partial}{\partial x} \right) \left(\frac{\partial u}{\partial \xi} \right)^2 - e^2 H_0^2 u/p \right] \quad (1.4) \end{aligned}$$

$$-\left(\frac{1}{p} \right) \left(\frac{p}{x} \right) + \left(\frac{v}{h} \right) \left(\frac{2v}{\eta} \right) + v^2 + \left(\frac{1}{p} \right) \left(\frac{p}{x} \right) + \left(\frac{v}{h} \right) \left(\frac{2v}{\eta} \right) + v^2$$

$$\begin{aligned} & \left[\left(\frac{2}{h} \right) \left(\frac{2}{\eta} \right) \left(\frac{v}{h} \right) \left(\frac{v}{\eta} \right) \right] + 2 \left(\frac{u}{x} \right) \left(\frac{u}{x} \right) \left(\frac{u}{x} \right) + \left(\frac{4}{h^2} \right) \left[\left(\frac{u}{x} \right) \left(\frac{2u}{\eta} \right) + \left(\frac{v}{h} \right) \left(\frac{2v}{\eta} \right) - \right. \\ & \left. \frac{2}{x} + \left(\frac{u}{x} \right) \left(\frac{2u}{\eta} \right) + \left(\frac{v}{h} \right) \left(\frac{2v}{\eta} \right) \right] \left[\frac{u}{x} + \left(\frac{v}{h} \right) \frac{u}{\eta} \right] \left[\frac{u}{x} + \left(\frac{v}{h} \right) \frac{u}{\eta} \right] + \left(\frac{v^3}{h^2} \right) \left[\frac{4}{\left(\frac{v}{\eta} \right)^2} + \left(\frac{u}{\eta} \right)^2 \right] \quad (1.5) \end{aligned}$$

$$k(2T/x^2 + 2T/y^2) + \rho c_v (uT/x + vT/y) + \rho c_v (uT/x + vT/y) + \rho c_v (uT/x + vT/y) + \rho c_v (uT/x + vT/y) + \rho c_v (uT/x + vT/y) + \rho c_v (uT/x + vT/y) \quad (1.6)$$

$\nu_1 (=1/\rho)$ is the kinematic viscosity, $\nu_2 (=2/\rho)$ is the kinematic elastic-viscosity, $\nu_3 (=3/\rho)$ is the kinematic coefficient of cross-viscosity, c_v is the specific heat at constant volume, k is the thermal conductivity, and $\eta = y/h$ is the dimensionless distance.

The viscous dissipation function is defined as follows:

$$= \rho \nu_1 \left(\frac{\partial u}{\partial x} \right)^2 + \rho \nu_2 \left(\frac{\partial v}{\partial y} \right)^2 + \rho \nu_3 \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)^2 \quad (1.7)$$

The mixed deviatoric stress tensor is denoted by τ_{ij} .

The boundary criteria are as follows:

$$\begin{aligned} (u/\eta)_{\eta=0} = 0, (v/\eta)_{\eta=0} = 0, (u/\eta)_{\eta=1} = 0, (v/\eta)_{\eta=1} = 0, (u/\eta)_{\eta=0} = 0, \\ v(x,0) = 0, v(x,1) = V, v(x,-1) = -V, v(x,-1) = -V \\ T(x, 1) = T_1, T(x, -1) = T_1 \quad (1.8) \end{aligned}$$

We get by substituting (1.2) in equations (1.4) and (1.5) and removing p from the resulting equation.

$$1 (f' v - f v') - S_2 f'' = 0, f' v + R(f' f'' - f f''') + 1 (f' v - f v') - S_2 f'' = 0, \quad (1.9)$$

The suction is represented by $R (= Vh/\nu_1)$. The Hartmann number, $S_2 = [eH_0 h / (1)]^{1/2}$, is an elastic-viscous parameter guiding the effects of elastic-viscosity of the fluid, and Reynolds number, $1 (= \nu_2 V / h \nu_1)$, is an elastic-viscous parameter governing the effects of elastic-viscosity of the fluid.

The shape of the temperature distribution is suggested by equations (1.6) and (1.2) as follows:

$$T = T_1 + (\nu_1 V) \left[\left(\frac{U}{V} - \left(\frac{x}{h} \right)^2 \right) \right] / (h C_v) \quad (1.10)$$

We derive the coefficient of $(U/V - x/h)^2$ and terms independent of $(U/V - x/h)^2$ on both sides of the resultant equation by using equation (1.10) in equation (1.6) and equating the coefficient of $(U/V - x/h)^2$ and terms independent of $(U/V - x/h)^2$ on both sides of the resulting equation.

The Prandtl number is $\rho = 1 c_v / k$, and the second-order parameter is $2 = 2 \nu_2 / (h^2 \rho)$.

The temperature distribution may be stated in a dimensionless manner as follows:

$$T = (T - T_1) / (T_1 - T_1) = E(+2), T = (T - T_1) / (T_1 - T_1) = E(+2), T = (T - T_1) / (T_1 - T_1) \quad (1.13)$$

The following is the solution to equations (1.18), (1.19), and (1.20) when the boundary condition (1.21) is applied:

$$f_0(\eta) = (1/2)(3-3)(1/2)(3-3)(1/2)(3-3)(1/2)(3-3)(1/2)(3-3)(1/2)$$

$$f_1(\eta) = -(1/280)(7-3\eta^2) - S_{12}/40(5-2\eta^2), \quad f_1(\eta) = -(1/280)(7-3\eta^2) - S_{12}/40(5-2\eta^2),$$

$$f_2(\eta) = (1/1293600)(14\eta^{11} - 385\eta^9 + 198\eta^7 + 876\eta^3 - 703) - (1/280) \quad f_2(\eta) = (1/1293600)(14\eta^{11} - 385\eta^9 + 198\eta^7 + 876\eta^3 - 703) - \{(3\xi^7 - 9\xi^3 + 6\xi) + S_{12}(\xi^7 - 3\xi^3 + 2\xi)\} - S_{12}(1/100800)(159 + 1087\eta - 947\eta^2 - 545\eta^3 - 2763\eta^4 + 207\eta^5) + (S_{12}/8400)(57 - 215\eta + 273\eta^2 - 11\eta^3) = 0(\eta) = 0(\eta) = 0(\eta) = 0(\eta) = 0 \quad P(1-\eta) = 1(\eta) = (3/2)P(1-\eta)$$

$$2(\eta) = 3P^2 383/280 - 85/6/10 + 4/4 - (3/2)^2 - P(9/280) (1-\eta)^2 + (S_{12}/10) (1 + 2\eta^6 - 34\eta^3) - (3/5)$$

$$P^2 (1-\eta)^6 \quad P^2 (1-\eta)^6 \quad P^2 (1-\eta)^6 \quad P^2 (1-\eta)^6 \quad P^2 (1-\eta)^6$$

$$(w/2)(\eta) = 1(\eta)$$

$$(10\eta^3 - 5\eta^2 - 9) - (P/2) (21\eta^2 + 6\eta - 64 - 16)$$

$$\phi_2(\xi) = P^2 [29\xi^{10}/840 - 51\xi^8/140 + 37\xi^6/20 - 9\xi^4/2 - 1149\xi^2/280 + (w/40) (1391/2520 - 93/2 + 995/20 - 157/14 + 59) - P[11/168 - 332/280 + 11\eta^4/140 - 36/140 - 38/280 + 10/168 - S_{12}(2\eta^2/5 - 13\eta/8 + 280 + 6/5 - 74/20 - 57/280) + 2(3 - 32/5 - 3\eta/8 + 10 + 12\eta^6/5 - 94/2) - w(71/100800 - 3/840 + 3\eta^5/5600 - 9/20160) + S_{12}(19/8400 - 7/1680 + 5/400 - 3/240)].$$

DISCUSSIONS AND RESULTS

- (i) The values derived by Sharma and Singh for the functions f_0 , f_1 , and f_2 are identical.
- (ii) The results for $2 = 0$ are quite similar to those obtained by Terril and Shrestha.
- (iii) The results for $S = 0$ are identical to those obtained by Agarwal.

CONCLUSIONS

The fluctuation of the temperature profile for $R = 0.01, 0.1, 1.0$ at $P = 0.4, S = 0.4, E = 1, S_1 = 1, 2 = -1$ shows that for $R = 0.1$, temperature climbs up to roughly $\eta = 0.7$ and then progressively falls until it reaches its value 1 at the boundary wall $\eta = 1$. The temperature graph is parabolic with the vertex upward at $R = 1$ and reaches its greatest value in the centre of the wall gap-length, with the minimum value at the border wall $\eta = -1$. Temperature rises linearly across the wall gap-length for $R = 0.01$, with a minimum at the boundary wall $\eta = -1$ and a high at $\eta = 1$. It is also obvious from this diagram that when the suction Reynolds number R increases,

the temperature rises. The temperature profile for $P = 0.4$, $\omega = 0.4$, $E = 1$, $S_1 = 1$, $R = 1$ for $\beta = 0, 0.1, 1.0$ shows that the temperature graph is generally parabolic with vertex upward and reaches its greatest value in the centre of the wall gap-length with a minimum at the border wall $\eta = -1$. This picture also shows that when the cross-viscous second-order parameter β increases, the temperature falls. The temperature graph is essentially parabolic with vertex upward and attains its greatest value at the centre of the wall gap-length with a minimum at the border wall $\eta = -1$ for $P = 0.4$, $\omega = 0.4$, $E = 1$, $R = 1$, $\beta = -1$ for $S_1 = 0, 1, 2$. This graphic also shows that when the Hartman number S_1 increases, the temperature lowers.

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