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Linear Algebra: A Way Defined for Image Processing and Networking

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Abstract:

Mathematicians who specialize in linear algebra study vectors, vector spaces (also known as linear spaces), mappings (sometimes referred to as linear transformations), and equation systems in linear form. Linear algebra and matrices are investigated in this work. A similarity matrix is used to investigate the connection between digital image processing and linear algebra, and problems with packet delivery involving linear algebra matrices are also addressed. Also, solve the traffic congestion by proposing Convolutional Neural Network-Convolutional-Long Short-Term Memory (CNN-ConvLSTM). The results provide the lowest Mean Squared Error (MSE) and Mean Absolute Error (MAE) values are 0.18521 and 0.15362 on weekdays, respectively. It is also provided the lowest MSE, and MAE values are 0.65650 and 0.90678 on weekends.

Keywords: Linear Algebra, Similarity matrix, CNN-ConvLSTM, MSE and MAE

Preliminaries

- Linear system: A linear algebra course would not be complete without solving a linear system. A Linear system is defined as a finite set of linear equations in a finite set of variables. For example, $x_1, x_2, \ldots, x_n$ or x, y, and z are all examples of linear equations. Linear algebra allows for the interpretation of a great number of problems in terms of linear systems, utilizing vector spaces and matrices.
- Vectors: An element of vector space is all a vector is, there is no other way to say it. A phenomenon that has two independent properties such as magnitude and direction, for example, gravity, magnetic field, acceleration, etc.

1. Introduction

Linear algebra is an area of mathematics that deals with vectors, vector spaces, and linear mappings (also called linear transformations). A central subject in modern mathematics is vector

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spaces, and linear algebra is frequently used in both abstract algebra and functional analysis [1]. The study of linear algebra demands familiarity with algebra, analytic geometry, and trigonometry [2]. Algorithms for doing image processing on digital images are developed by Digital Image Processing (DIP). Students who are interested in learning more about linear algebra can benefit from using mathematics as a teaching method. [3-4]. A vector is a segment of a directed line whose magnitude, is represented by its length, and direction. To represent physical elements like forces, the first real vector space can be constructed by adding and multiplying scalars and vectors. The scope of contemporary linear algebra has been broadened to include the investigation of spaces with arbitrary or infinite dimensions [5-6]. Cartesian 2- and 3-space vectors were at the root of linear algebra's origins in the field [2].

The field of mathematics known as linear algebra is concerned with linear equations like as,

$$a_1x_1 + a_2x_2 + \cdots + a_nx_n = b \qquad (1)$$

Linear maps, for example,

$$(x_1, x_2, \ldots, x_n) \mapsto a_1x_2 + a_2x_2 + \cdots + a_nx_n, \qquad (2)$$

All branches of mathematics can be traced back to linear algebra. In equations 1 and 2, $x_1, x_2, \ldots, x_n$ are variables, and $a_1, a_2, \ldots, a_n$, b are real or fixed values.

An n-space is a vector space with n dimensions. These higher dimensional spaces can be used to expand most of the good results from 2- and 3-space. Vectors and n-tuples are useful for describing data, even if people have a hard time seeing them in n-space. This paradigm makes it feasible to effectively summarize and handle data by vectors, in their capacity as n-tuples are ordered lists of n components. The Gross National Product (GNP) of eight separate countries can be represented, for example, using 8-dimensional vectors or 8-tuples in economics. A year-by-year comparison of the gross domestic product of eight different countries might be useful, for example (the United States, the United Kingdom, France, Germany, Spain, India, Japan, and Australia), and vectors $v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8$ can be used, with the GNP of each country in its appropriate position [1].

DIP fascinating educational tool is presented to supplement and enhance the current linear algebra course [7-8]. Color, brightness, contrast, and saturation are all components of a digital image, which is a discrete representation of data. Image processing is a method of improving or extracting useful information from an image by conducting a range of operations on the image. Signal processing is applied to images, and the result is a new image or other features that are connected to the image [9]. The field of image processing is rapidly expanding. Imaging encompasses several phases such as the importation of images, the analysis and manipulation of images, and finally the output of images that have been altered or reports based on image analysis. In the case of printouts and images, analog image processing can be applied. Computers are used in digital image processing to modify images. Pre-processing, augmentation and display, and information extraction are three of the main stages of digital processing. Improving pictorial information and process images for data storage, transfer, and representation for autonomous machine perception are two of DIP's key aims [7]. Students can be assisted in their understanding of linear algebra using images. Many students are unaware that creating image files necessitates the use of a variety of mathematical instruments. A matrix can be used to represent a digital image stored on a computer [10-11].

1.1 Linear Equations

The study of equations led to the development of algebra. There are many ways to approach this problem, such as looking for all real values that are equal to $x = x^2 - 6$. For the solution, rewrite the equation as, $x^2 - x - 6 = 0$, and then factor in its left-hand side, which tells us that (x − 3) (x + 2) = 0. As a result, researchers would infer that either x = 3 or x = 2 must be true because neither x - 3 nor x + 2 can be true. The search for the roots of a polynomial is a nonlinear problem, whereas the theory of linear equations is the focus of this area of research [12-13].

Linear operators and optimization are used extensively in signal and image processing, remote sensing, and inverse problems. Existing software applications can construct dense or sparse matrices explicitly and perform algebraic operations with syntax that nearly matches. Their equivalent mathematical notation for small to medium-sized problems (for example, Python, NumPy, SciPy, and Matrix Laboratory (MATLAB)) [14-15-16-17]. Multi-industry use of the matrix is common. Matrices would be employed as the fundamental mathematical tool, to create a realistic animation from a polygon figure. As video games became more popular, computer graphics became increasingly prevalent. Thus, matrix multiplication serves as the foundation for programming a 3-Dimensional (3D) video game [1].

1.2 Matrix

One definition of a matrix describes it as a structured collection of integers that has a specified number of rows and columns.

Example: $[r_1\ r_2\ r_3]$ - row matrix, $\begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix}$ - column matrix

$$\begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{bmatrix} \tag{3}$$

1.2.1 Matrix Equation

Ax = B is a matrix equation, where A is a $m \times n$ matrix, B is a R^m vector, and x is a vector with unknown coefficients $x_1, x_2, \ldots, x_n$. Mathematical solutions to matrix equations are needed in a wide range of computer graphics challenges. Mathematical matrix formulations of problems are frequently encountered in graphics, thus first placing this field at the very top of my to-know list.

$$x_1 \begin{bmatrix} a_{11}\ a_{12} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{n1}\ a_{n2} & \cdots & a_{nn} \end{bmatrix} = \begin{bmatrix} a_{11}x_1 + a_{12}x_2 + & \cdots & +a_{1n}x_n\ a_{12} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \quad \vdots & \ddots & \vdots \\ a_{n1}x_1 + a_{n2}x_2 + & \cdots & +a_{nn}x_n\ a_{n2} & \cdots & a_{nn} \end{bmatrix} \tag{4}$$

1.2.2 Multiplication property of matrix

Matrix multiplication is only defined if the number of columns in each of the two matrices involved is equal to the number of rows. A is an m-by-n matrix and B is an n-by-p matrix; the matrix product of these two matrices, called AB, is an m-by-p matrix whose entries are given by AB.

$$[AB]_{i,j} = A_{i,1}B_{1.j} + A_{i,2}B_{2,j} + \cdots + A_{i,n}B_{n,j} = \sum_{r=1}^{n} A_{i,r}B_{r,j} \tag{5}$$

Here, i = elements of rows and j = elements of columns.

where $1 \leq j \leq p$ and $1 \leq i \leq m$. For example (the highlighted entry 1 in the product is computed as the product $1 \times 1 + 2 \times 2 + 3 \times 0 = 5$):

$$\begin{bmatrix} 1 & 2 & 3 \\ -4 & 6 & 2 \end{bmatrix} \times \begin{bmatrix} 0 & 1 \\ 1 & 2 \\ 5 & 0 \end{bmatrix} = \begin{bmatrix} 17 & 5 \\ 16 & 8 \end{bmatrix} \tag{6}$$

It is true that matrix multiplication follows the laws of (AB)C = A(BC), and that it also follows the rules of (A+B) C = AC+BC, as well as C(A+B) = CB+CA when the matrices are large enough to define the various products. AB can be defined without defining BA if A and B are m-by-n and n-by-k matrixes, individually, with the property that $m \neq k$. Matrix multiplication, in contrast to (rational, real, or complex) numbers whose product is unaffected by the order of the elements, is not commutative, even if both products are specified; in general, one has $AB \neq BA$. [5].

2. Related Work

This section shows the several related works of many authors which has described below.

Magalhães et al., (2022) [18] stated the Monte Carlo-based distributed linear algebra. Random walks over a system's matrix are used to determine the system's matrix powers, which must subsequently be utilized to compute a specified matrix function using the approach. To handle bigger problems, the matrix must be distributed among several nodes, however, that imposes a communication cost since walks must be compelled to hop across computing nodes. Buffering solutions have been investigated and researchers have found a solution that reduces the cost and optimizes performance. Researchers applied their approach to the computation of network metrics like node centrality and the resolvent Estrada index. Researchers show that their distributed implementation can successfully solve previously inaccessible problem cases on extremely large networks.

Gharrawi et al., (2022) [19] studied the Weighted Least Squares (WLS) method to determine the most suitable position for a portable drone station that provides services related to traffic management. Applying the WLS method to the daily traffic congestion data of Kansas City, Missouri. Researchers showed that a polygonal limited drone station position was optimal using this method of presentation. This can be done by relocating the drone station to a better location depending on data acquired during morning and afternoon peak hours. Inside the context of smart city planning. This research highlights the necessity of WLS optimization techniques.

Ahmadi et al., (2021) [20] suggested that a Convolutional Neural Network (CNN) is used to segment the tumors found in glioma, meningioma, Alzheimer's, and Alzheimer's plus, Pick, Sarcoma, and Huntington's disease. Researchers began by analyzing a dataset provided by the Harvard Medical School and used the feature-reduction strategy of robust principal component analysis to pinpoint the location and size of the tumor. Researchers describe the architecture of a CNN approach for detecting brain cancers. Magnetic Resonance Imaging (MRI) scans were used to estimate the likelihood of tumor localization. Other studies have shown good accuracy (96 percent), 99.9 percent sensitivities, and 91% dice index for the approach described. Both the suggested supervised architecture and the suggested unsupervised approach for tumor clustering have medical applications.

Zhang et al., (2021) [21] investigated optimizing congestion systems using the network congestion game. The semi-tensor product of matrices is initially used to build the network congestion game matrix equation. Network congestion games can only work if there is a pre-requisite and adequate condition. It is possible to design the price of traffic congestion using an algorithm. Once again, individuals can maximize their utility function using well-designed learning rules to improve transportation networks. As a result, the findings are more accurate and richer because of two special circumstances. Finally, the approach was used to highlight the utility of the findings.

Chen et al., (2019) [22] stated a new Low-Rank Quaternion Approximation (LRQA) model. Instead of the standard sparse representation and Low-Row Matrix Approximation (LRMA)-based approaches, a pure quaternion matrix is used to hold the color image. LRQA limits the rank of the built-in quaternion matrix. The singular values of a low-rank quaternion matrix can be estimated more correctly using a broad LRQA model based on several nonconvex functions. Sparse models from LRQA outperform the best in the industry. Additionally, LRMA-based methods for quantitative evaluations and visual quality in color picture denoising and inpainting difficulties.

Bayar et al., (2018) [23] investigated a novel technique for use in forensic investigations that makes use of convolutional neural networks (CNNs). CNNs could learn classification features directly from the data they are fed, but in their current state, they tend to learn characteristics suggestive of the content of a photo. To solve this issue, the authors have designed a new type of CNN layer known as a limited convolutional layer. Image modification detecting features are suppressed while this layer learns to do so adaptively. Researchers conducted a series of experiments to demonstrate that the suggested constrained CNN can learn characteristics for identifying manipulations. At 99.97 percent accuracy, the experiments show that CNN exceeds the current best general-purpose manipulation detection. As a result, the restricted CNN was able to accurately recognize picture changes in real-world circumstances when the training and testing data's source camera model differed.

Jin et al., (2018) [24] suggested a biomedical imaging-focused deep convolutional network for inverse problems. The suggested technique, known as filtered back projection (FBP) ConvNet, combines FBP with a CNN with several resolutions. The design of CNN is based on U-net, but with the addition of residual learning. This method was motivated by the convolutional structure of different biomedical inverse problems, such as computed tomography (CT), magnetic resonance imaging (MRI), and diffraction tomography (DT) scans. Specific restrictions on linear operators guarantee that their normal operators are also convolutions. U-net is at the heart of CNN's design, which also integrated residual learning. CT, MRI, and DT inverse issues all have a convolutional structure that inspired this technique. Linear operators have been shown to have requirements that assure their normal operator is a convolution.

Li et al., (2018) [25] suggested that Basic Linear Algebra Subprograms (BLAS), can categorize a wide range of matrices and provide a consistent interface. Central Processing Unit (CPU) and Graphics Processing Unit (GPU) are two of the most extensively used heterogeneous computing platforms today. For the time being, BLAS can be used on both the CPU and the GPU. A unique matrix approach might be built for a certain processor because of the varying properties of algorithms and hardware. It is essential to conduct any computation involving a matrix using the appropriate processor. The BLAS is followed by an introduction to CPU and GPU architecture and optimization strategies. BLAS's various subroutines were explored experimentally for their effects. Finally, researchers explore the motivations for matrix computations as well as the processor selection scheme.

Shehab et al., (2018) [26] suggested a fragile watermarking approach based on Single Valued Decomposition (SVD) that utilized the grouped block strategy to provide an additional layer of protection as well as a means of locating the compromised regions within the many medical photographs. There are two types of authentication bits that can withstand the vector quantization attack such as block confirmation and self-recovery bits. Improved Normalized Cross-Correlation (NCC) and Peak Signal-to-Noise Ratio (PSNR) could be achieved by employing Arnold's transform to recover the tampered region. The findings of the experiments

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demonstrated that the strategy that was given is quite reliable and can locate the blocks that have been attacked. Researchers intend to find a solution to this problem in their upcoming study. In addition, the researchers would concentrate their efforts on identifying additional types of image manipulation, such as resizing, skewing, and rotating procedures.

Defferrard et al., (2016) [27] stated GSP approaches could be used to effectively adapt CNNs to graphs employing mathematical and computational underpinnings. Graph convolutional layers have been used to illustrate the model's ability to extract local and stationary properties. According to the model, researchers have greater control over the local support of filters than in previous work on spectral graph CNNs. The model was also computationally more efficient because it doesn't explicitly employ the Graph Fourier basis. Future studies would look at two other avenues. On the one hand, researchers would use newly developed GSP tools to supplement the recommended framework. Research would next focus on applying this general model to important sectors where data naturally occur on graphs, which might also contain external information about the structure of the data, rather than artificially creating graphs whose quality may fluctuate as observed in the tests.

Mou et al., (2015) [28] investigated a distributed method for finding an exponentially fast solution to a linear equation that is solvable if the required and sufficient conditions are met. It's possible that the equation could only have one viable solution in that case. Some questions have been left unresolved by the researchers for future investigation. The first step is to determine the nature of the connection that exists between the parameter A, which might be found in a conditioning number of A and the convergence rate bound B. Alternatively, if A and B's variations are time-varying, it is possible to measure the link between those variations and the tracking error e more precisely. Alternatively, the least-squares technique presented in the study could be modified to limit the quantity of information transmitted between agents.

Chetlur et al., (2014) [29] suggested a library of deep learning primitives known as cuDNN. Using highly optimized matrix multiplication techniques, the authors developed a novel version of convolutions that is dependable over a wide range of input sizes and does not require additional memory. The authors also provide a set of techniques that allow users to train and evaluate complete deep neural networks without having to write parallel code by hand. In the future, the machine learning company would benefit greatly from these libraries. If using multiple GPUs to speed up training, they plan on using this library.

Narang et al., (2013) [30] intended a new approach to graph-structured data interpolation that draws on technologies from signal processing. The authors defined the interruption question as the refurbishment of a graph indicator from a downsampled-upsampled (DU) tasted sign. Results so far reveal a superior balance between precision and complication when associated with other existing algorithms. Future study must first define the essential and sufficient needs for reconstruction (the hypothesis given in the paper was just a sufficient condition), and this will only be possible by first establishing the essential and sufficient conditions for reconstruction.

2.1 Comparison of Review Literature

A summary of related work is described in Table 1. There is a wide range of authors who used the technique and presented their discoveries, as given in the table.

Table 1. Summary of Related Work.

Authors	Technique used	Outcomes
Magalhães et al., [2022] [18]	The Monte Carlo-based distributed linear algebra solver	Distributed implementation can successfully solve previously inaccessible problem cases on extremely large networks
Gharrawi et al., [2022] [19]	The WLS method	This research highlights the necessity of WLS optimization techniques.
Ahmadi et al., [2021] [20]	CNN	Research has shown good accuracy with 99.9 percent sensitivities, and a 91% dice index for the approach described
Zhang et al., [2021] [21]	Optimizing congestion systems using the network congestion game	The findings are more accurate and richer because of two special circumstances.
Chen et al., [2019] [22]	LRQA model	Singular values of a low-rank matrix can be estimated more accurately using this model based on its noisy data
Bayar et al., (2018) [23]	A novel technique for use in forensic investigations	The restricted CNN was able to accurately recognize picture changes in real-world circumstances
Jin et al., (2018) [24]	A biomedical imaging-	Linear operators have been shown

	focused deep convolutional network	to have requirements that assure their normal operator is a convolution
Li et al., [2018] [25]	Basic linear algebra subprograms (BLAS)	As a result, to keep within the power and thermal limit, standard CPUs can only fit a restricted number of processor cores on the same die
Shehab et al., [2018] [26]	A fragile watermarking approach based on SVD	The findings of the experiments demonstrated that the strategy that was provided is quite dependable and can effectively locate the blocks that have been assaulted.
Defferrard et al., (2016) [27]	GSP approaches	The model was also computationally more efficient because it doesn't explicitly employ the Graph Fourier basis
Mou et al., [2015] [28]	A distributed algorithm	Researchers can measure the link between those variations and the tracking error e more precisely
Chetlur et al., (2014) [29]	cuDNN	The results allow users to train and evaluate complete deep neural networks without having to write parallel code by hand
Narang et al., (2013) [30]	A new approach to graph-structured data interpolation	Results so far revealed a superior balance between precision and complication when associated with other existing algorithms

3. Background Study

An essential topic in university mathematics, linear algebra focuses on matrix theory and the combination of matrices, finite-dimensional vector space, and linear transformation theory. Mathematical symbols define linear algebra principles, rather than importing examples. The first step is to let students see the course as a part of their everyday lives when teaching a difficult and abstract course. Matrix for investigating the application in record sales and traffic difficulties in the workplace. Linear algebra and digital image processing are studied by researchers. Researchers have discovered that digital image editing uses matrix operations. The purpose of this research is to go over some of the basic principles of linear algebra that are utilized in the field of computer graphics. It is now possible to see how mathematics, and linear algebra, can be used in engineering. People's daily life has been significantly affected by traffic congestion in

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recent years. Using fine-grained traffic forecasts for individual urban road segments can help commuters plan their paths in progress and assist traffic flow authorities in minimizing traffic congestion. Spatiotemporal traffic congestion data is generated using the CNN-ConvLSTM spatial matrix and fed into ConvLSTM for the extraction of spatiotemporal information and improved prediction performance [26].

4. Problem Formulation

One or more variables are used in linear equations where one variable is dependent on another. A linear equation can be used to depict almost any situation in which there is an unknown number, such as calculating income over time, computing mileage rates, or forecasting profit. The use of linear equations is common for most people. Linear systems are used to model a wide variety of problems such as traffic congestion and related to the sales record. Congestion in the metropolitan area is one of the most serious challenges of daily life. Linear algebra matrices are used in the work for detecting traffic congestion, which is explained by the one-way route between two places problem. CNN-ConvLSTM is suggested in this research as a spatiotemporal traffic prediction model to provide short-term predictions for each road segment's congestion level Congestion propagation patterns and geographical correlations between road segments should be incorporated into a spatial matrix. Packet delivery is also a major problem in daily life, when a factory sends the product to many stores at a time, sellers faced a lot of problems. A solution to the packet delivery problems by using linear algebra matrices is also discussed in the paper.

5. Research Methodology

Linear algebra is used in many places in daily life. In this research, linear algebra is used to define image processing via a similarity matrix. in traffic congestion to solve traffic problems. The work also focused on sales record problems such as packet delivery. A solution with the help of matrix multiplication is also discussed by researchers to solve sales problems.

5.1 Object detection via Similarity Matrix for Image Processing

Matrix observation is used in face recognition programs. In this step, an image of a person is entered into the software, and the program creates a face matrix based on that image. The software can recognize the consistency of patterns in a person's face when the matrix is

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generated. Among the various ways, matrix theories are used in image processing for face detection is the Eigenface technique [7].

A pixel is the smallest determinable component of a digital image. An image's atom is represented by a pixel. Between a 24-bit and a 32-bit system can be found in many computers. An image's quality improves as the number of pixels increases. Color blending results in pixel coloration. Red, blue, and green are the three primary colors that are used to create all colors (called the Red, Green, and Blue (RGB) model). Three base colors have 256 distinct shades. The value of the red, green, and blue components determines the color of a pixel. This indicates that with the RGB system, researchers can construct 256 * 256 * 256 = 16777216 different sorts of pixels. The image is automatically written in the RGB system as a matrix whose entries are the values of the pixels. As a result, every image may be linked to a matrix and vice versa. MATLAB can transform images into matrices and matrices into images [7].

As soon as it is determined that a digital image may be represented by matrices, researchers can inquire about the effects of operations performed on the elements of the associated image.

Suppose the binary image X, as a matrix [19], i.e.,

$$X = (a_{i.j}), \qquad (7)$$

Here, X = matrix form of a binary image, $a_{i.j}$ = elements of X in form row (i) and column (j).

Therefore, the picture is equivalent to the transposed matrix of X, which is,

$$B = (b_{i,j}) = (a_{j,i}) = X = X^T, \qquad (8)$$

B = X^T = transpose image of X.

An image H corresponds to the matrix $(a_{j,35-i+1})$, to return to the example.

In digital image processing, the multiplication of matrices is also used. Linear algebra university studies are required to understand the reasoning behind the next example, which uses more complex mathematical approaches. Researchers believe that the ability to decompose a matrix as the product of special-structure matrices would enable the development of an amazing application. Now consider the singular value decomposition (SVD), which is the product of three matrices structures written as the matrix $A_{m\times n}$,

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$$A_{m\times n} = U_{m\times n} S_{m\times n} V_{n\times n}^T \, , \qquad (9)$$

where U and V are orthogonal matrices (that is, $U^T U$ and $V^T V$ are $m \times m$ and $n \times n$ identity matrices, respectively) and S is a matrix whose elements $S_{i,j} = 0$, for $i \neq j$ and $S_{1,1} \geq S_{2,2} \geq \ldots \geq S_{k,k} \geq 0$, with $k = min\,\{m, n\}$. An example of an SVD decomposition,

$$A = \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} = USV^T \qquad (10)$$

$$= \begin{bmatrix} -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix} \begin{bmatrix} \sqrt{3} & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \frac{\sqrt{6}}{6} & -\frac{\sqrt{2}}{2} & \frac{\sqrt{3}}{3} \\ -\frac{\sqrt{6}}{3} & 0 & \frac{\sqrt{3}}{3} \\ \frac{\sqrt{6}}{6} & \frac{\sqrt{2}}{2} & \frac{\sqrt{3}}{3} \end{bmatrix}^T \qquad (11)$$

The SVD decomposition of every matrix may be demonstrated. Furthermore, computer methods exist that can be used to perform such decompositions. As researchers can see from their example, the most important thing to keep in mind is that the columns of matrix U are represented by $u_1, u_2, \ldots\ldots u_m$, and the columns of matrix V are represented by $v_1, v_2, \ldots\ldots v_m$, then [7]

$$A = USV^T = s_{1,1} u_1 v_1^T + s_{2,2} u_1 v_2^T + \ldots\ldots + s_{k,k} u_k v_k^T. \qquad (12)$$

5.2 Linear Algebra for detecting Traffic Congestion and Packet Delivery

Researchers might find solutions to a wide variety of real-world issues by using the matrix because of its intimate connection to life. The usage of matrices in this project is to investigate applications in the areas of record sales, traffic congestion, people movement, and energy consumption. The application of life knowledge must also be used to remind students at the same time. There has been a great deal of focus on traffic congestion in recent years because of the significant negative impact it has on people's everyday lives. Individual road segment fine-grained congestion forecast in urban areas to relieve traffic congestion by helping commuters to plan their journeys. A congestion prediction model must be accurate since road segment granularity is so important [31]. Most traffic jams happen during rush hour or because of one-time events like parties, large-scale business promotions, protests, traffic controls, accidents, or parades. Traffic management systems can profit from this in several ways, including avoiding and clearing traffic congestion at the proper moment, as well as generating suitable proposals for the future growth of traffic networks [32-33]. The monitoring of traffic flow is essential for traffic management and safety enhancements. A real-time traffic status forecast could also benefit greatly from it [34-35].

Example 1: A one-way route between the two places is depicted in Figure 1 using a matrix representation of the route [26].

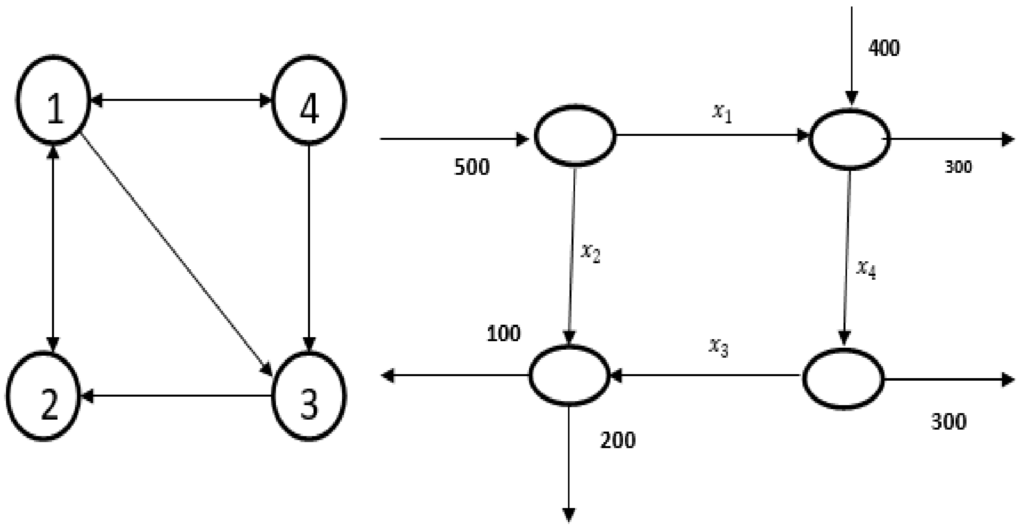

Figure 1. Traffic map.

$$\text{Set } a_{ij} = \begin{cases} 1, & from\ the\ city\ i\ to\ j\ has\ a\ one-way\ route \\ 0, & from\ the\ city\ i\ to\ j\ no\ one-way\ route \end{cases}$$

The one-way route between four cities, shown as a matrix, is as follows,

$$A = (a_{ij}) = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \end{bmatrix}, \tag{13}$$

For example, many points between the channels are readily available according to the principle, i.e.,

$$A^2 = (b_{ij}) = \begin{bmatrix} 2 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \end{bmatrix}, \tag{14}$$

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The number of people who have traveled from city i to city j via a single route is expressed in b. In the illustration that can be found above, the value $b_{23} = 1$ indicates that the number of people that traveled from City 2 to City 3 by a single route is 1. (2→1→3)

$b_{42} = 2$ displays the number 2, which represents the number of people who traveled from City 4 to City 2 by a single route. (4→1→2), (4→3→2)

$b_{11} = 2$ is displayed, and the number of two-way routes in city 1 is displayed.

After the display of city 1 is 2, there is not a two-way route for the $b_{33} = 0$.

Example 2: [26] A matrices can be used to represent a factory's ability to distribute four different types of products to three different retailers,

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix}, \qquad (15)$$

Go to the i store on behalf of the a_{ij} and send the j products there. For the matrix form, one may also use unit price and weight for each of the four products,

$$B = \begin{bmatrix} b_{11} & b_{21} & b_{31} & b_{41} \\ b_{21} & b_{22} & b_{32} & b_{42} \end{bmatrix}^T, \qquad (16)$$

The first i items are represented by b_{ij}, which gives the unit price and the first i products are represented by b_{i2}, which gives the single weight. To calculate $AB = C = (c_{ij})_{3\times2}$, researchers need to use the matrix that was produced by the express gross to three retailers and the total weight. Researchers multiply A by B using the definition of matrix multiplication, which implies that c_{i1} represents the first i store's gross product and c_{i2} reflects first i store's total product weight.

Example: [23] Table 2 depicts the product weights, pricing, and quantities that were delivered to three retailers by a manufacturing company.

Table 2. Sales Sheet.

	Air conditioner	**Ice Box**	**29″ color TV**	**25″ color TV**
A Shop	30	20	50	20
B Shop	0	7	10	0

C Shop	50	40	50	50
Unit Price	30	16	22	18
Single weight	40	30	30	20

As a result, the factory's plan to ship four products to three retail locations can be stated as a matrix,

$$A = \begin{bmatrix} 30 & 20 & 50 & 20 \\ 0 & 7 & 10 & 0 \\ 50 & 40 & 50 & 50 \end{bmatrix}, \tag{16}$$

It is also possible to utilize the price and weight of each of the four products as a matrix representation,

$$B = \begin{bmatrix} 30 & 16 & 22 & 18 \\ 40 & 30 & 50 & 20 \end{bmatrix}^T. \tag{17}$$

This is the product of multiplying matrices A and B, which represents the matrix generated by the express gross to the three stores and the total weight,

$$AB = C = (c_{ij})_{3\times 2} \tag{18}$$

$$AB = \begin{matrix} A\ shop \\ B\ shop \\ C\ shop \end{matrix} \begin{bmatrix} totalprice & totalweight \\ 2680 & 3700 \\ 332 & 510 \\ 4140 & 5700 \end{bmatrix}. \tag{19}$$

6. Numerical Result

This section will introduce the data used in the experiments and compare CNN-ConvLSTM with six different models.

6.1 Convolutional LSTM

Convolutional LSTM (ConvLSTM) extends LSTM by incorporating convolutions into the learning process. Like the hybrid model of CNN and LSTM, ConvLSTM uses direct convolutions as input to LSTM units. Here are the main ConvLSTM equations. For example, convolutions can be added to an LSTM model to make it more flexible. ConvLSTM differs from the CNN/LSTM hybrid model in that it incorporates convolutions directly into the LSTM units during the reading process. ConvLSTM's primary equations are displayed as follows:

$$i_t = \sigma(W_{xi} * X_t + W_{hi} * H_{t-1} + W_{ci} \circ C_{t-1} + b_i) \tag{20}$$

$$f_t = \sigma(W_{xf} * X_t + W_{hf} ** H_{t-1} + W_{cf} \circ C_{t-1} + b_f) \quad (21)$$

$$C_t = f_{t-1} \circ C + i_t \circ tanh(W_{xc} * X_t + W_{hc} * H_{t-1} + b_c) \quad (22)$$

$$o_t = \sigma(W_{xo} * X_t + W_{ho} * H_{t-1} + W_{co} \circ C_t + b_o) \quad (23)$$

$$H_t = o_t \circ tanh\,(C_t) \quad (24)$$

where i_t, o_t, f_t, represent the timestamp t's outputs from the input gate, output gate, and forget gate, in that order. C_t, H_t are the cell outcomes and unknown state of the cell at time move t, the $*$ and $\circ$, while and represent the convolution process and Hadamard creation, individually, in this example.

6.2 Performance Metrics and Baseline Method

Mean Squared Error (MSE) and Mean Absolute Error (MAE) are two metrics that are utilized in the task for performance evaluation.

$$MSE = \sum_n MSE = \frac{\sum_j^N \left(\frac{1}{S} \sum_i^S (y_i - \bar{y}_i)\right)^2}{N} \quad (25)$$

$$MAE = \sum_n MAE = \frac{\sum_j^N (\frac{1}{S} \sum_i^S |y_i - \bar{y}_i|)}{N} \quad (26)$$

Here, y_i and $\bar{y}_i$ are road segment's actual and anticipated congestion levels.

S = total number of road segments for i.

N = total number of road segments j.

In the equations y_i and $\bar{y}_i$ a road segment's actual and anticipated congestion levels can be found. The entire number of congestion propagation (CP) road segments in a traffic congestion propagation pattern graph is equal to the total number of S road segments in the graph. Prediction accuracy is demonstrated by a low MSE and MAE.

6.3 Counterparts

In this section, CNN-ConvLSTM is compared to the following six counterparts.

- Long-term time series models such as LSTM can be used to anticipate congestion levels at the segment level, rather than using a global average. A two-layer LSTM is used to make the time series prediction.

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- CNN-LSTM: A two-layer LSTM and a two-dimensional CNN are employed to extract spatial and temporal data, respectively. The fusion model can make predictions based on a grid map by merging CNN and LSTM. In the same way as the LSTM method above, it is predicting at the road segment level.
- CP-CNN-LSTM: Incorporating only the proposed CPs improves CNN-LSTM.
- ConvLSTM: Congestion propagation pattern graphs and spatial matrices are omitted from the convLSTM 2D model, which is only built from a grid map. Prognoses are broken down by route segment in this scenario, as well.
- CNN-ConvLSTM: Using spatial matrices to depict congestion propagation patterns improves CNN-LSTM.

6.4 Performance Comparison

In the first part of this section, describe the main results of the six approaches given in Table 3. CNN-ConvLSTM had the lowest MSE and MAE in both weekday and weekend data [33].

Table 3. Comparison table of performance of different models.

Models	**Weekends**		**Weekdays**	
	MAE	**MSE**	**MAE**	**MSE**
LSTM	0.303783	0.317636	0.444490	0.598624
CNN-LSTM	0.715469	1.139783	0.993106	2.103802
ConvLSTM	0.653928	0.977357	0.898181	1.736250
CP-CNN-LSTM	0.419031	0.467619	0.54024	0.837977
CPM-CNN-LSTM	0.228535	0.166550	0.315467	0.358728
CPM-CovnLSTM	0.228535	0.075692	0.187260	0.270992
Proposed CNN-ConvLSTM	0.90678	0.65650	0.15362	0.18521

Figure 2 shows the Performance comparison on partial congestion graphs (weekdays) as below:

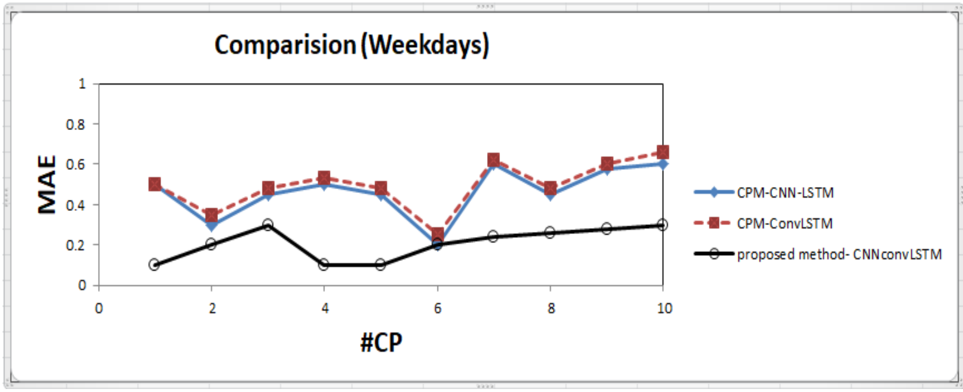

Figure 2. Performance comparison on partial congestion graphs (weekdays)

Figure 3 shows the Performance comparison on partial congestion graphs (weekends) as given below:

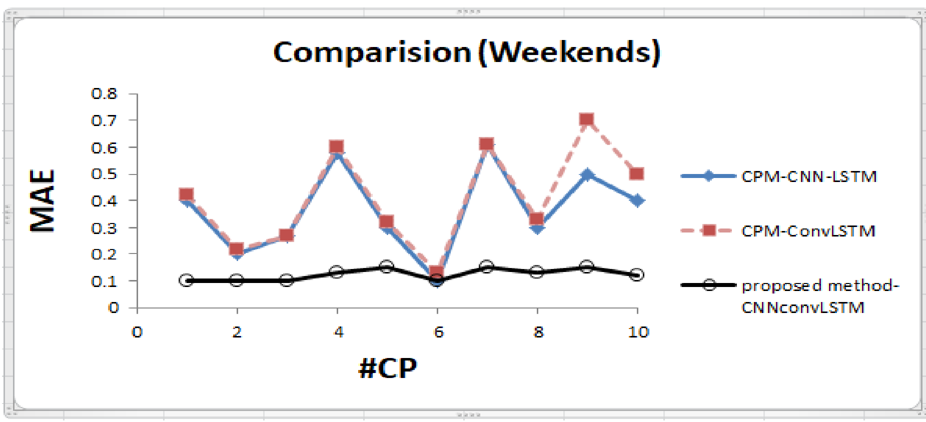

Figure 3. Performance comparison on partial congestion graphs (weekends)

LSTM outperforms CNN-LSTM and ConvLSTM in terms of accuracy. LSTM can forecast independently for each side road section, however, CNN-LSTM and ConvLSTM predicts the graininess of grid cells concerning a grid cell's average level of congested roadways as an indicator. The average approach yields a higher error rate than the LSTM method, which is not difficult to determine. The results of ConvLSTM are consistent with the projection that ConvLSTM will perform better than CNN-LSTM.

Error rates increase as the accuracy of CNN-LSTM and ConvLSTM improves; this is true for CP-CNN-LSTM and CP-CP. Using SpaMat (spatial matrix) to encode the CP, CPM-CNN-LSTM, and CNN-ConvLSTM can further reduce CNN-LSTM and ConvLSTM's error rates. For CNN-LSTM and ConvLSTM, SpaMat is a valuable tool for improving their performance so far.

It randomly selects ten of the discovered CPs and examines how four different techniques perform on each of the CPs. Even on the weekends, CNN-ConvLSTM consistently has the lowest error. Besides, the mistakes of three additional techniques vary widely on different CPs.

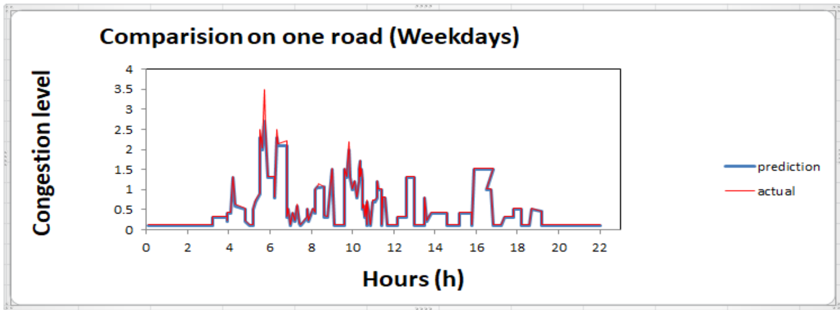

Figure 4. Comparison of actual and predicted congestion (weekday)

For every hour of every day, plot the predicted congestion levels in Figure 4. According to the graph, CPM-predicted ConvLSTM's congestion levels for this road section are very similar to the actual congestion levels. A last look at CNN-ConvLSTM MSE and MAE is made by adjusting the period duration.

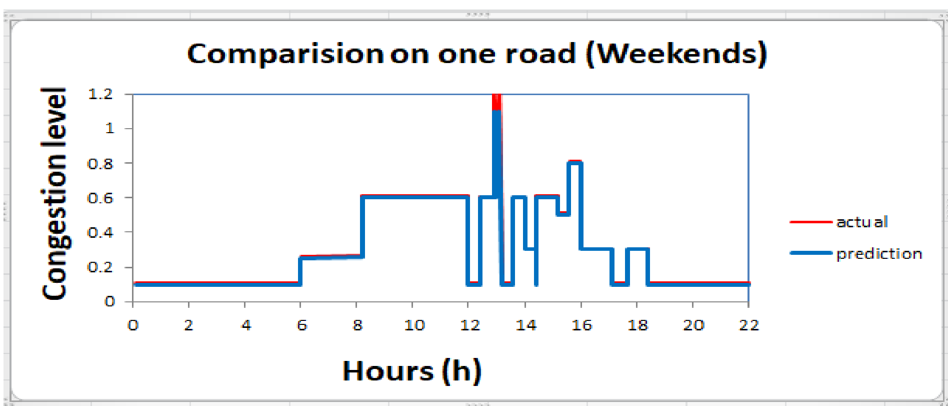

Figure 5. Comparison of actual and predicted congestion (Weekends).

Using a randomly selected road stretch, the projected congestion levels are then depicted in Figure 5 for each hour of the day. On this specific stretch of road, the CNN-ConvLSTM forecast result closely matches the amount of traffic congestion. Researchers found that increasing the time difference led to higher mistake rates, ranging from 1 to 15 minutes. Having less time to collect the data means that CNN-ConvLSTM will have the most up-to-date and accurate congestion information.

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7. Conclusion and future work

Algebraic expressions with linear equations are those in which each component has only one constant or one variable multiplied. Record sales and image processing detection could benefit from the use of similarity matrices, according to a new study. The CNN-ConvLSTM model for traffic congestion prediction is also described in this study. Road segment congestion levels can be predicted using the aging of congestion propagation patterns. The key impact of the research is the use of a CNN-ConvLSTM (spatial matrix) to combine congestion propagation patterns and the physical proximity of road sections. From the table 3, initial results show that CNN-ConvLSTM greatly outperforms the current state-of-the-art in terms of prediction errors on real-world data. The results provide the lowest MSE, and MAE values are 0.18521 and 0.15362 on weekdays, respectively. It is also provided the lowest MSE, and MAE values are 0.65650 and 0.90678 on weekends. It is hoped that in the future, scientists will be able to explore more complex patterns of congestion propagation that consider both spatial and temporal information. External factors such as weather and the surrounding environment will also be considered by researchers to improve prediction accuracy.

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