
DETERMINATION OF THE FOCAL LENGTH OF A CONVERGING LENS – CASE STUDY AT KADE SENIOR HIGH TECHNICAL SCHOOL USING TWO LENSES LABELLED $F = 20\text{CM}$ AND 15CM IN EASTERN REGION OF GHANA – AFRICA.

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Abstract

Focal length determination of converging lenses is a major question by WEAC and a major problem for its determination by both students and physics teachers in some SHS in Ghana during this international examination. This research work sorts to investigate two lenses labelled $f = 20\text{cm}$ and 15cm by the suppliers and bought by the school as directed by the examining body for the physics practical. The investigation will determine by practical determination of focal length to verify the value quoted by the supplier and given to students to use as the focal length of the lens. The investigation also sort to justify the practical procedure and understanding of students. The methods employed in this work to determine f includes the uv method, far and near distance method and the Bessel's method. The far and near method gave focal lengths for the 20cm and 15cm converging lens as $(19.70 \pm 0.24)\text{ cm}$ and $(14.88 \pm 0.09)\text{ cm}$ respectively. FL values of $(20.12 \pm 0.15)\text{ cm}$ and $(15.04 \pm 0.08)\text{ cm}$ is the experimental values obtained for the 20cm and 15cm converging lenses using the uv method. The Bessel's method gave experimental FL values of $(19.74 \pm 0.32)\text{ cm}$ and $(14.98 \pm 0.04)\text{ cm}$ for the 20cm and 15cm after the investigation. This gives a clear indication that, all the converging lenses supplied by suppliers had the focal lengths correctly quoted. One major identified problem with the practical procedure is the use of the far and near method to obtain the focal length (FL) value of the converging lens and then justifying the value obtained through the whole experimental process with the Bessel's method. An approximate focal length value of 19.5cm can be obtained at a distance of 800cm (8m) and this can't be said is done by students hence unjustifiable. An exact value of 20cm is obtained at infinity which is far beyond 800cm and therefore questionable as expected of students.

Key words: focal length (FL), lens, converging lens, uv method, Bessel's method, WAEC, WASSCE.

1 INTRODUCTION

1.1 Convex lens and image formation

A ray of light passing through a glass lens may be refracted twice - first where it enters the glass, and then where it leaves the glass. There is a net deviation of the ray from its original direction. Usually each surface of the lens is part of a sphere. Lenses with two spherical surfaces sufficiently close together that the distance between them (the thickness of a lens) can be neglected are called thin lenses. If their thickness cannot be neglected, they are called thick lenses. A converging lens is capable of focusing a beam of parallel light into a small region at a focal point. A converging lens is any lens thicker in the center than at the edges (convex surface). Focal length is the distance between the focal point and the center of the lens (Fig 1).

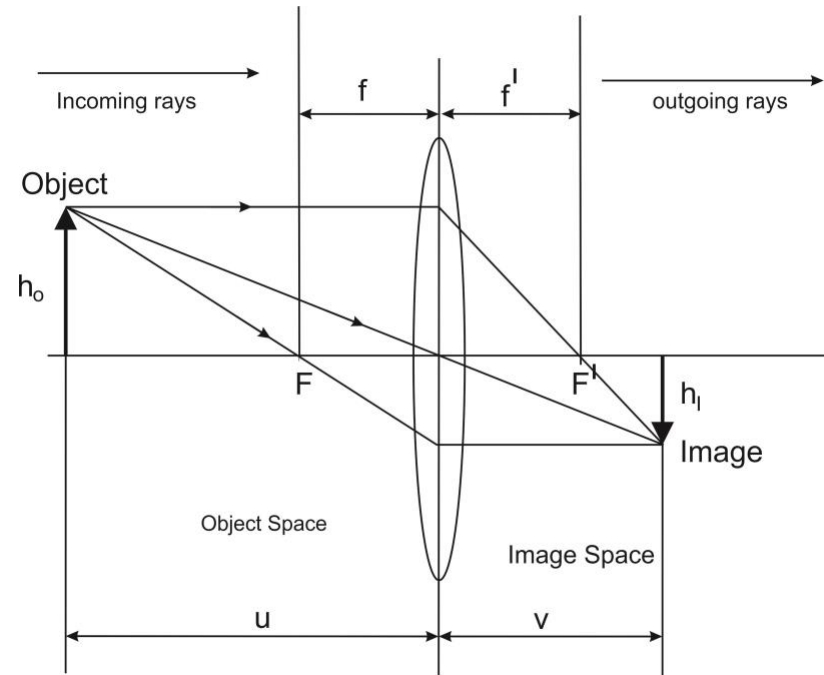


Fig 1: Diagram showing object and image formation in a convex lens

The following symbols are introduced;

u = the object distance

v = image distance

f = first (near) focal length

f' = Second (far) focal length

h_o = height of object

h_i = height of image.

Three principal rays are usually drawn to find an image (**Fig. 1**).

- A ray parallel to the optical axis, after refraction by the lens, passes through the second focal point.
- A ray through a center of the thin lens is not deviated.
- A ray through the first focal point emerges parallel to the optical axis

1.2 The far and near Method

The ability of a lens to focus light is a consequence of its shape and optical density relative to that of the surrounding environment. The purpose of this experiment is to investigate the focal length of the converging lens used at WASSCE for its accuracy as indicated on the package and used by teachers and students using the far and near method.

Because of its ability to focus light, a converging lens can produce an image of a far and near object, as represented in Fig.2. The object and image are denoted by the line segments h_o and h_i . The red lines are a representative sample of light rays emanating from point B. The lens redirects and focuses the light rays at point B'. In this example the image is characterized as real and inverted. It is real because a viewer to the right of the lens would observe light rays originating from the image, as is shown for the image point B'.

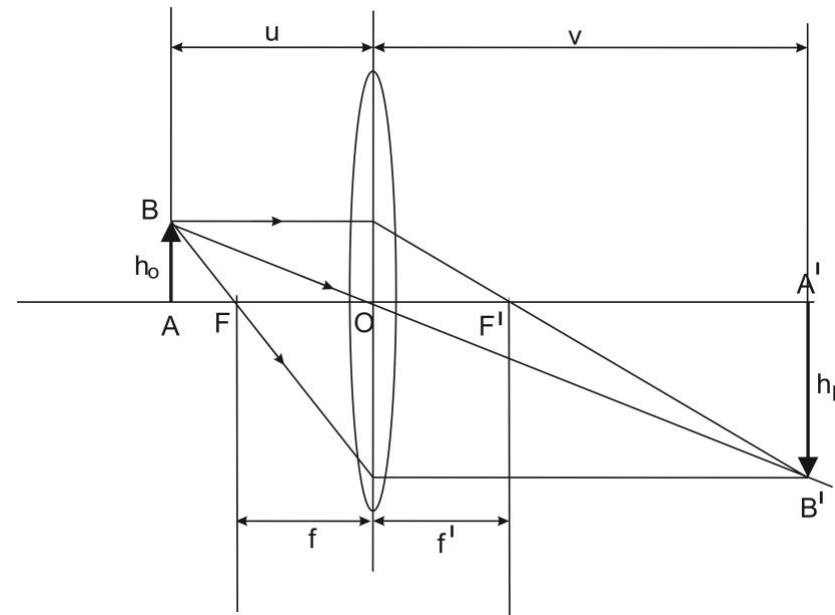


Fig. 2: Converging lens producing real inverted image

The focal points on either side of the lens are denoted F and F'. Whenever the two spherical caps have the same curvature, as in Fig. 2, the distance from the lens to either focal point is the same. Thus, there is a single focal length denoted f which equals the distance $OF = O'F'$. One can show that the focal length is related to the following quantities,

$$f = \frac{uv}{v + u} \dots \dots \dots (1)$$

Where the object distance u equals AO , and the image distance v equals OA' . In Equation 1 taking the limit as u approaches

infinity, which corresponds in practice to locating the object far from the lens, one obtains;

$$f = \lim_{u \rightarrow \infty} \frac{uv}{v + u}$$

$$f = \lim_{u \rightarrow \infty} \frac{u(v)}{u(\frac{v}{u} + 1)}$$

$$f = \lim_{u \rightarrow \infty} \frac{v}{\frac{v}{u} + 1}$$

$$f = \frac{v}{\frac{v}{\infty} + 1}$$

$$f = v \dots \dots \dots (2)$$

This is the case for taking the object from far. Showing that light coming from a distant source is focused at the focal point of the lens. Hence the focal length of the lens is equal to the distance between the lens and the screen. Since the image is formed on the screen as seen in Fig. 2.

In the case of taking objects very close to the lens, the image will be produced far away from the lens, hence the image distance will be a huge value hence approaching infinity. In that case we have the following equation;

$$f = \lim_{v \rightarrow \infty} \frac{uv}{u + v}$$

$$f = u \dots \dots \dots (3)$$

Equation 3 indicates that, for an object located very close to the lens will produce an image at infinity given a focal length of the converging lens equal the distance between the object and the converging lens.

1.3 The uv Method

The uv method employed in this experimental verification is determining u and v experimentally to determine the focal length of the lens. Consider fig 2 above;

u = object distance

v = image distance and

f = focal length of the converging lens.

The lens formula in Gaussian form is given by

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v} \dots \dots \dots (4)$$

This implies that,

$$f = \frac{uv}{v + u} \dots \dots \dots (5)$$

This is same as equation 3.

To perform this experiment, turn on the lamp in the ray box and move the lens along the optical bench until you get a clear image of the object on the screen. Measure the distances u_1 and v_1 . Calculate the focal length using the lens formula as

indicated in equation 5. The procedure is repeated for the experiment for about four times, and in each case the corresponding value of f evaluated. The mean value of the focal length (f) is calculate and error analysis is performed. The final result is obtained is written as $f = \bar{f} \pm \sigma$, where σ is the standard deviation.

Result obtained is again drawn graphically with uv as ordinate and $v + u$ as abscissa. The slope of the graph is equal to the focal length of the converging lens. That is,

$$\text{Slope} = f = \frac{\Delta(uv)}{\Delta(v + u)}$$

This is repeated for about three different lenses of the same focal length as given for the examination for its accuracy in using to obtain correct results by students.

1.4 The Bessel's Method

Bessel's method utilizes the measurement of two lens positions which produces a near and far image at the same image position for an object at a fixed position. In practice, this involves fixing the positions of the object and viewing screen and varying the position of the lens until two distinct lens positions are found which produce a clear distinct image on the screen. As long as $L > 4f$, we are guaranteed to find two positions which produce an image at the viewing of the screen. The experimental setup defines the relationships presented in 6 and 7, where L is the distance between the object and the

viewing screen and d is the distance between the two lens positions (A and B) which results in clear image formation on the screen. The thin lens equation applied to the lens at position one and position two resolves into equations 8. (Uladzimir, 2017). Analyzing the experimental setup along with equations 6 and 7 we can conclude that the object distance in lens position one is equal to the image distance in lens position two, $u_1 = v_2^l$, and vice versa, $u_2 = v_1^l$, therefore reducing equations 8 and 9 into equations 10 and 11 respectively.

$$u_1 + v_1^l = u_2 + v_2^l = L \dots \dots \dots (6)$$

$$u_2 - u_1 = v_1^l - v_2^l = d \dots \dots \dots (7)$$

$$f = \frac{u_1 v_1^l}{v_1^l + u_1}$$

$$f = \frac{u_1 v_1^l}{L} \dots \dots \dots (8)$$

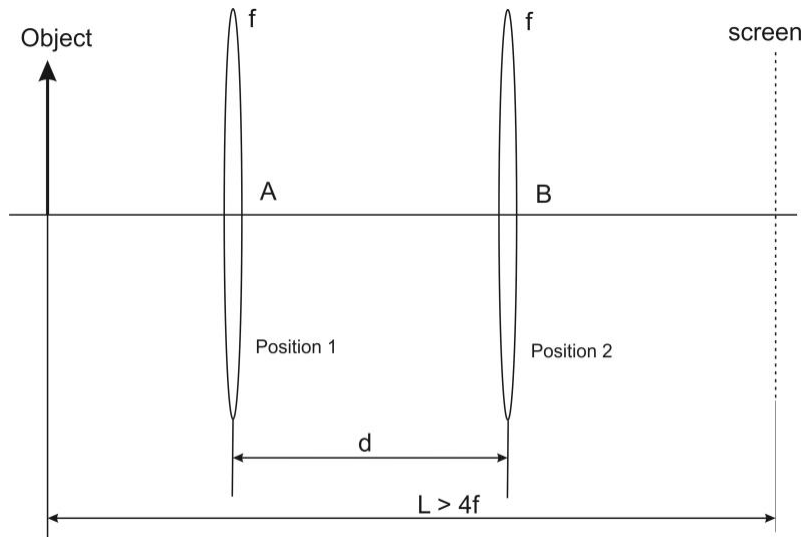


FIG. 3. Lens Positions and Notation for Bessel's Method

$$u_2 + u_1 = L \dots \dots \dots (9)$$

$$f = \frac{u_1 u_2}{u_2 + u_1}$$

$$f = \frac{u_1 u_2}{L} \dots \dots \dots (10)$$

Adding and subtracting equations 10 and 7 results in equations 11 and 12 respectively. Multiplying equations 11 and 12 then resolves into a relationship between the distances between the two lenses, d, the distance between the object and image L, and the focal length, f, as demonstrated in equation 13.

$$2s_2 = L + d \dots \dots \dots (11)$$

$$2s_1 = L - d \dots \dots \dots (12)$$

$$4s_2 s_1 = 4fL = (L + d)(L - d) = L^2 - d^2 \dots (13)$$

$$f = \frac{L^2 - d^2}{4L} \dots \dots \dots (14)$$

Equation 14 is therefore the formula for the determination of the focal length of a converging lens by Bessel's.

2 EXPERIMENTATION PROCEDURE

The three methods presented above were utilized to assess the focal lengths of five different lenses of the same focal length type given as 15cm and 20cm by the supplier. Lens systems consisted of one converging lens of different focal lengths (i.e. one given as 15cm focal length lens and other as 20cm focal length lens). A lens system consisting of the convex lenses as directed by WAEC for WASSCE were then analyzed utilizing the focal length determination equation for each method. Five different lenses of the same focal lengths $f = 20\text{cm}$ and $f = 15\text{cm}$ were selected for the verification process.

3 RESULTS AND DISCUSSIONS

3.1 Far and Near Method

3.1.1 For the focal length $f = 20\text{cm}$

Lens No.	u/cm	v/cm	1/u(cm)	1/v(cm)	f/cm
1	20	600	0.05	0.00167	19.4

2	20	700	0.05	0.00143	19.4
3	800	20	0.00125	0.05	19.5
4	200	22	0.005	0.04550	19.8
5	5000	20	0.0002	0.05	19.9
6	∞	20	0	20	20

$$\bar{f} = \frac{(19.4 + 19.4 + 19.5 + 19.8 + 19.9 + 20)cm}{6}$$

$$\bar{f} = 19.70cm$$

3.1.2 For the focal length f = 15cm

Lens No.	u/cm	v/cm	1/u(cm)	1/v(cm)	f/cm
1	20	60	0.05	0.017	14.9
2	20	60	0.05	0.017	14.9
3	20	58	0.05	0.017	14.9
4	57	20	0.018	0.005	14.7
5	20	60	0.05	0.017	14.9
6	∞	15	0	15	15

$$\bar{f} = \frac{(14.9 + 14.9 + 14.9 + 14.7 + 14.9 + 15)cm}{6}$$

$$\bar{f} = 14.88cm$$

3.2 uv Method

3.2.1 For the focal length f = 20cm

Lens No.	u/cm	v/cm	uv/cm	(u + v)/cm	f/cm
1	30	62.5	1875	92.5	20.3
2	30	63	1890	93.0	20.3
3	30	60	1800	90.0	20.0
4	30	60	1800	90.0	20.0
5	30	60	1800	90.0	20.0

$$\bar{f} = \frac{(20.3 + 20.3 + 20.0 + 20.0 + 20.0)cm}{5}$$

$$\bar{f} = 20.12cm$$

3.2.2 For the focal length f = 15cm

Lens No.	u/cm	v/cm	uv/cm	(u + v)/cm	f/cm
1	30	30	900	60	15.0
2	30	30	900	60	15.0
3	30	31	930	61	15.2
4	30	30	900	60	15.0
5	30	30	900	60	15.0

$$\bar{f} = \frac{(15.0 + 15.0 + 15.2 + 15.0 + 15.0)cm}{5}$$

$$\bar{f} = 15.04cm$$

3.3 Bessel's Method

3.3.1 For the focal length f = 20cm

Lens No.	L/cm	d/cm	L ² /cm ²	d ² /cm ²	f/cm
1	150	104.5	22500	10920.25	19.3
2	150	102.5	22500	10506.25	20.0
3	150	102.5	22500	10506.25	20.0
4	150	102.5	22500	10506.25	20.0
5	150	104	22500	10816.00	19.4

$$\bar{f} = \frac{(19.3 + 20.0 + 20.0 + 20.0 + 19.4)cm}{5}$$

$$\bar{f} = 19.74cm$$

3.3.2 For the focal length f = 15cm

Lens No.	L/cm	d/cm	L ² /cm ²	d ² /cm ²	f/cm
1	150	116.0	22500	13456.00	15.0
2	150	116.0	22500	13456.00	15.0
3	150	116.0	22500	13456.00	15.0
4	150	116.5	22500	13572.25	14.9
5	150	116.0	22500	13456.00	15.0

$$\bar{f} = \frac{(15.0 + 15.0 + 15.0 + 14.9 + 15.0)cm}{5}$$

$$\bar{f} = 14.98cm$$

3.4 Focal length error analysis

3.4.1 Far and Near Method error analysis

3.4.1.1 For the focal length f = 20cm

f/cm	\bar{f} /cm	$(f - \bar{f})$ /cm	$(f - \bar{f})^2/cm^2$
19.4	19.70	-0.3	0.09
19.4	19.70	-0.9	0.09
19.5	19.70	-0.2	0.04
19.8	19.70	0.1	0.01
19.9	19.70	0.2	0.04
20.0	19.7	0.3	0.09
			$\sum (f - \bar{f})^2 = 0.36$

$$\sigma = \sqrt{\frac{\sum (f - \bar{f})^2}{N}}$$

$$\sigma = \sqrt{\frac{0.36}{6}}$$

$$\sigma = 0.24cm$$

3.4.1.2 For the focal length f = 15cm

f/cm	\bar{f} /cm	$(f - \bar{f})$ /cm	$(f - \bar{f})^2/cm^2$
14.9	14.88	0.02	0.0004
14.9	14.88	0.02	0.0004
14.9	14.88	0.02	0.0004
14.7	14.88	-0.18	0.0324
14.9	14.88	0.02	0.0004
15.0	14.88	0.12	0.0144

	$\sum (f - \bar{f})^2 = 0.0484$
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$$\sigma = \sqrt{\frac{\sum (f - \bar{f})^2}{N}}$$

$$\sigma = \sqrt{\frac{0.0484}{6}}$$

$$\sigma = 0.09\text{cm}$$

3.4.2 uv Method Error analysis

3.4.2.1 For the focal length f = 20cm

f/cm	\bar{f}/cm	$(f - \bar{f})/\text{cm}$	$(f - \bar{f})^2/\text{cm}^2$
20.3	20.12	0.18	0.0324
20.3	20.12	0.18	0.0324
20.0	20.12	-0.12	0.0144
20.0	20.12	-0.12	0.0144
20.0	20.12	-0.12	0.0144
			$\sum (f - \bar{f})^2 = 0.108$

$$\sigma = \sqrt{\frac{\sum (f - \bar{f})^2}{N}}$$

$$\sigma = \sqrt{\frac{0.108}{5}}$$

$$\sigma = 0.15\text{cm}$$

3.4.2.2 For the focal length f = 15cm

f/cm	\bar{f}/cm	$(f - \bar{f})/\text{cm}$	$(f - \bar{f})^2/\text{cm}^2$
15.0	15.04	-0.04	0.0016
15.0	15.04	-0.04	0.0016
15.2	15.04	0.16	0.0256
15.0	15.04	-0.04	0.0016
15.0	15.04	-0.04	0.0016
			$\sum (f - \bar{f})^2 = 0.032$

$$\sigma = \sqrt{\frac{\sum (f - \bar{f})^2}{N}}$$

$$\sigma = \sqrt{\frac{0.032}{5}}$$

$$\sigma = 0.08\text{cm}$$

3.4.3 Bessel's Method Error analysis

3.4.3.1 For the focal length f = 20cm

f/cm	\bar{f}/cm	$(f - \bar{f})/\text{cm}$	$(f - \bar{f})^2/\text{cm}^2$
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19.3	19.74	-0.44	0.1936
20.0	19.74	0.26	0.0676
20.0	19.74	0.26	0.0676
20.0	19.74	0.26	0.0676
19.4	19.74	-0.34	0.1156
			$\sum (f - \bar{f})^2 = 0.512$

$$\sigma = \sqrt{\frac{\sum (f - \bar{f})^2}{N}}$$

$$\sigma = \sqrt{\frac{0.512}{5}}$$

$$\sigma = 0.32cm$$

3.4.3.2 For the focal length f = 15cm

f/cm	\bar{f}/cm	$(f - \bar{f})/cm$	$(f - \bar{f})^2/cm^2$
15.0	14.98	0.02	0.0004
15.0	14.98	0.02	0.0004
15.0	14.98	0.02	0.0004
14.9	14.98	-0.008	0.0064
15.0	14.98	0.02	0.0004

	$\sum (f - \bar{f})^2 = 0.008$
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$$\sigma = \sqrt{\frac{\sum (f - \bar{f})^2}{N}}$$

$$\sigma = \sqrt{\frac{0.008}{5}}$$

$$\sigma = 0.04cm$$

3.5 Experimentally determined focal length (FL) values:

Based on the focal length determination for each of the methods used and the error analysis, the final focal length is obtained as $f = \bar{f} \pm \sigma$. Using the far and near method, the focal lengths obtained for the 20cm and 15cm FL lens are (19.70 ± 0.24) cm and $((14.88 \pm 0.09)$ cm respectively. FL values of (20.12 ± 0.15) cm and (15.04 ± 0.08) cm is the experimental values obtained for the 20cm and 15cm converging lenses recommended by WAEC to be used by students during the practical when the uv method was used. The last method which is the Bessel's method gave experimental FL values of (19.74 ± 0.32) cm and (14.98 ± 0.04) cm for the 20cm and 15cm defined focal lengths (FL) as given by West African Examination Council (WAEC) for the practical examination.

3.6 Determined focal length (FL) value correspondence to that given by WEAC

Three methods namely the far and near method, the uv method and the Bessel’s method were the three main methods used in this research in verifying for the focal lengths of the recommended converging lenses used for the physics practical. The sampled (West African Senior School Certificate Examination) WASSCE questions were WASSCE 2021 physics 3 practical Alt A question two and WASSCE 2021 physics 3 practical Alt B question two. The three methods used to obtain the FL values gave correct corresponding acceptable values when compared to the focal lengths of the 20cm and 15cm converging lenses asked by WAEC to be used for the practical examination. The main aim of the WASSCE 2021 physics 3 practical Alt B question two is to determine the focal length of the converging lens used for the practical. This is justified by plotting a graph with $D^2 - d^2$ on the vertical axis and D on the horizontal axis. By this;

$$s = \frac{D^2 - d^2}{D} \dots \dots \dots (15)$$

Where s is the slope of the graph.

A parameter called k is evaluated as $k = \frac{s}{4} \dots \dots \dots (16)$

The k value determined here is equal the f which is the focal length of the converging lens used for the practical. Substituting equation (16) into (15) gives;

$$k = \frac{D^2 - d^2}{4D} \dots \dots \dots (17)$$

Comparing equation (14) and (17), $k = f$, which is the focal length of the converging lens, where $L = D$, which is the distance between the object and the screen.

This cannot be seen in the case of the WASSCE 2021 physics 3 practical Alt A question two which when compared to WASSCE 2021 physics 3 practical Alt B question two, follows the same procedure and is based on the Bessel’s method for the determination of the FL value of the converging lens used for the practical.

3.7 Far and near method used for determining the FL – value

Both the WASSCE 2021 physics 3 practical Alt A question two and WASSCE 2021 physics 3 practical Alt B question two, sub question (i) have the same statement and answer; ‘Determine the approximate focal length, f , of the lens by focusing a distant object on the screen’. By this question, students are tasked to first of all determine the focal length of the converging lens being used for the practical and then justified the answer as seen in equation (17). This is a justification for WASSCE 2021 physics 3 practical Alt B question two but can’t be said same for WASSCE 2021 physics 3 practical Alt A question two.

By the question above, and answer especially for the 20cm focal length lens, students can never determine the focal length of the lens by focusing a distance object within the environs of the laboratory for all the schools within which the practical is being conducted. To get the exact FL value of 20cm, the distance object should be at infinity (*equation 2*) and to get an approximate value within range, the focusing object should be at distance of 800cm. Below this value, no other FL value can be obtained to justify for the 20cm focal length converging lens used for the practical. This is illustrated using the prove for equation (2).

$$f = \lim_{u \rightarrow 800} \frac{uv}{v + u}$$

$$f = \frac{20}{\frac{20}{800} + 1}$$

$$f = 19.5cm$$

At infinity,

$$f = \lim_{u \rightarrow \infty} \frac{v}{\frac{v}{u} + 1}$$

$$f = \frac{20}{\frac{20}{\infty} + 1}$$

$$f = 20.0cm$$

Therefore $u < 800cm$, will not give justifiable FL values for the converging lens used for the practical. Students are not in a position of determining the FL of the converging lens at this value let alone its justification using the Bessel's formula. Therefore is the need for defining the distance at which the approximate value of the focal length of converging lens used for the practical should be determined.

3.8 FL – Value exactness estimation as determined by the different methods to equate to the WAEC FL value

Focal length value exactness estimation looks at the values of the focal length of the converging lens obtained experimentally using the three methods and its sameness with the given FL values as 20cm and 15cm by WEAC for the WASSCE. The far and near method gave focal lengths for the 20cm and 15cm converging lens as (19.70 ± 0.24) cm and (14.88 ± 0.09) cm respectively. Comparing the given focal length of the converging lens of 20cm and 15cm to the experimentally obtained values, they are approximately 20cm and 15cm when the far and near method is used in their determination. FL values of (20.12 ± 0.15) cm and (15.04 ± 0.08) cm is the experimental values obtained for the 20cm and 15cm converging lenses using the uv method. The uv method also gave values of approximately 20cm and 15cm when compared to the 20cm and 15cm focal length converging lens respectively. The Bessel's method gave experimental FL values of (19.74 ± 0.32) cm and (14.98 ± 0.04) cm for the 20cm and

15cm after the investigation. The Bessel's method also gave equivalent values when compared to the focal length of the converging lens authorized by WAEC for the 2021 WASSCE.

4 Conclusion

Focal length investigation and validation for correspondence to that authorized by West African Examination Council (WAEC) is by the three methods; the far and near method, uv method and the Bessel's method. The far and near method gave focal lengths for the 20cm and 15cm converging lens as (19.70 ± 0.24) cm and (14.88 ± 0.09) cm respectively. FL values of (20.12 ± 0.15) cm and (15.04 ± 0.08) cm is the experimental values obtained for the 20cm and 15cm converging lenses using the uv method. The Bessel's method gave experimental FL values of (19.74 ± 0.32) cm and (14.98 ± 0.04) cm for the 20cm and 15cm after the investigation. After assessment and error analysis, all the focal lengths of the converging lens obtained by the three methods corresponded to the 20cm and 15cm authorized by WAEC for the WASSCE 2021 physics 3 practical Alt A and B.

Both the WASSCE 2021 physics 3 practical Alt A question two and WASSCE 2021 physics 3 practical Alt B question two, sub question (i) have the same statement and answer; '*Determine the approximate focal length, f , of the lens by focusing a distant object on the screen*'. By this question, students are tasked to first of all determine the focal length of the converging lens

being used for the practical and then justified the answer as seen in equation (17). This is a justification for WASSCE 2021 physics 3 practical Alt B question two but can't be said same for WASSCE 2021 physics 3 practical Alt A question two after critical assessment of the question.

In determining the focal length of the converging lens used for the practical, a distant object is expected to be focused onto the screen and f determined. This is unjustifiable in obtaining the approximate focal length value to be 20cm or 15cm (focal length of used converging lens) after focusing as distance is to be determined by each individual student. Hence the accuracy of the procedure and its justification with the final value of k for the WASSCE 2021 Physics 3 practical Alt B. Therefore $u < 800$ cm, will not give justifiable FL values for the converging lens used for the practical. Students are not in a position of determining the FL of the converging lens at this value let alone its justification using the Bessel's formula. Therefore is the need for defining the distance at which the approximate value of the focal length of converging lens used for the practical should be determined.

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