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THE PROBLEM OF DYNAMICS AND STABILITY OF ROD PROTECTED FROM VIBRATIONS IN RANDOM PARAMETRIC EXCITATIONS

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Abstract.

In this paper, the problem of dynamics and stability of nonlinear vibrations of a rod with elastic dissipative characteristic of the hysteresis type in conjunction with a liquid section dynamic absorber under the influence of random parametric excitations is considered. In this case, the system of differential equations of motion is reduced to the form of a system of Ito equations using the method of stochastic averaging. From the condition that the system of motion differential equations has a non-zero solution, the system characteristic equation is formed, the stability conditions are obtained and analyzed. The boundaries of the stability and unstability are expressed analytically, which allows to determine the average quadratic values of the vibrations of the rod protected from vibrations and to fully analyze the dynamic nature.

Introduction.

Taking into account the hysteresis type elastic dissipative characteristics of mechanical systems under the influence of various excitations used in modern techniques and technologies, the problems of their protection from harmful vibrations, study of dynamics and verification of stability are urgent problems. A great deal of theoretical and experimental research has been done on the mathematical modeling, study of dynamics, and exploration of the stability of mechanical systems. However, there is a need to conduct research to improve research methods and identify new mechanical effects that occur in the process, taking into account the elastic dissipative characteristics of hysteresis type of mechanical systems.

The work [1] developed the theoretical basis for mathematical modeling of complex motions of deformable solids using the method of bond graph. The kinetic and potential energies of the system are expressed by the variables of the bond graphs (impulses and velocities), and as a result the Lagrange second-order equations are given relative to these variables.

In the work [2], the vibrations of wind generators were mathematically modeled by the method of bond graph, and the advantages of this method are given. Vibration was used in a 20-sim program for numerical analysis of motion. The time-dependent variation of the first and second vibrations forms was quantified.

The work [3] deals with the problem of structural modeling of distributed systems with linear elastic characteristics by the method of bond graph. The equations of motion are expressed by first-order derivatives, and the stability of motion from the system matrix is explored. In this case, a characteristic equation is generated from the system matrix and the stability condition is determined by showing that the real part of the roots is negative.

In the work [4], the motions of rods and structures under the influence of external forces were mathematically modeled using the method of bond graph. In the 20-sim program, they are structurally expressed and numerically analyzed for amplitude-frequency characteristics. The vibration frequency ranges corresponding to the largest amplitudes are shown.

The article [5] explores the stability of the mean square value of systems under the influence of random parametric excitations using numerical methods. The Ito differential equation was generated by the stochastic averaging method, and it was shown that the results obtained for this averaged stability of system motion are also valid for a given stability of system motion. The analytical expression of stability condition of the system motion was determined depending on the damping coefficient and the spectral density of the excitation. Quantitative analysis of changes in stable and unstable fields depending on the damping coefficient.

In the work [6], the systems of differential equations of motion obtained by the method of averaging were analyzed. Not only the first approximation equations but also the second approximation equations were generated and the nonlinear vibrations of the systems were studied. Stationary solutions were found and their stability was explored. Solutions to some of the practical problems encountered in engineering have been obtained, numerically analyzed, and conclusions drawn.

The work [7] studied nonlinear free and forced vibrations of discrete and continuous systems. The systems were mathematically modeled using the Lagrange equation and the vibrations under the influence of random excitations were analyzed. An analysis of the Duffing equation was also performed, and the graph of the amplitude-frequency characteristic change in the nature of the phenomenon with hardening and softening of the spring is visualized. Stable and unstable fields are shown. The analytical expression of the vibration period was determined depending on the system parameters. Theoretical information on the basic relationships of random processes is given.

In the work [8], the stability of motion of nonlinear systems was theoretically studied in relation to changes in system parameters. Theorems on the separation of stable and unstable fields of system motion are given. The stability of motion of systems represented by a non-homogeneous differential equation with periodic coefficients of motion is analyzed.

The work [9] solves the problem of finding and analyzing the solution of the Ito differential equation for Weiner process. The condition that the solution of the Ito differential equation does not change abruptly is defined for non-autonomous system parameters.

In the work [10], the parametric stability of the motion of systems in real noisy excitations was studied. The Lyapunov exponent was identified and analyzed numerically (based on Maple15).

The work [11] studies the exponential stability of systems with hysteresis type connections under the influence of random excitations. The Ito differential equation was constructed using the stochastic averaging method and the Lyapunov method was used to explore the stability of motion. Several problems have been addressed in order to demonstrate the reliability of the results obtained.

The work [12] provides the theoretical basis of the method of bond graph and mathematical modeling of discrete and distributed parametric systems. In particular, the transverse vibrations of dynamic absorber, a hysteresis type elastic dissipative characteristic rod are mathematically modeled. In addition, a number of models representing the nonlinearity characteristics of systems have also been mathematically modeled by the method of bond graph, which are expressed analytically.

The analysis of the above work shows that one of the current problems of mechanics is the study of the dynamics and stability of transverse vibrations of a rod with elastic dissipative characteristics of the hysteresis type in conjunction with a liquid section dynamic absorber under the influence of random parametric excitations. Therefore, research in this area is currently required.

Materials and methods.

In this paper, the study of the dynamics and stability of transverse vibrations of the rod with an elastic dissipative characteristic of the hysteresis type in conjunction with liquid section dynamic absorber under the influence of random parametric excitations is considered.

The differential equations of motion of the rod protected from vibrations under consideration were obtained in the work [13] using the method of bond graph [14]:

$$A\ddot{Q} + B\dot{Q} + CQ = F, \tag{1}$$

where

$$\ddot{Q} = \begin{bmatrix} \ddot{q}_{i*} \\ \ddot{q}_3 \\ \ddot{q}_4 \end{bmatrix}; \dot{Q} = \begin{bmatrix} \dot{q}_{i*} \\ \dot{q}_3 \\ \dot{q}_4 \end{bmatrix}; Q = \begin{bmatrix} q_{i*} \\ q_3 \\ q_4 \end{bmatrix}; \ F = \begin{bmatrix} u_i(0)F_L + u_i(L)F_R \\ 0 \\ 0 \end{bmatrix};$$

$$A=\begin{bmatrix} m_i & 0 & 0 \\ (m_{13}+m_{2*})u_i(x_1) & m_{13}+m_{2*} & m_{2*}+m_v \\ (m_{2*}-m_v)u_i(x_1) & m_{2*}-m_v & m_{2*}+m_{4*} \end{bmatrix};$$

$$B=\begin{bmatrix} 0 & -u_i(x_1)b_f & 0 \\ 0 & b_f & 0 \\ 0 & 0 & b_S \end{bmatrix}; \quad C=\begin{bmatrix} c_i & -u_i(x_1)c_{1*} & 0 \\ 0 & c_{1*} & 0 \\ 0 & 0 & 2c_{2*} \end{bmatrix};$$

q_{i*} are displacements of the *i*-point of the rod; q_3 is displacement of the shell of the liquid section dynamic absorber surrounding the liquid; q_4 is displacement of solid of liquid section dynamic absorber in the liquid; $u_i(0)$, $u_i(L)$ and $u_i(x_1)$ are the values of the mode shapes of the rod at the left, right ends and at the points where the liquid section dynamic absorber is installed, respectively; F_L and F_R are the external forces applied to the left and right ends of the rod, respectively; m_i and c_i are modal mass and stiffness, respectively; $m_{13}=m_{1*}+m_{3*}$; m_{1*} is the inertial dimension of the body surrounding the liquid; m_{2*} is the inertial dimension (mass) of the body surrounded by the liquid; m_{3*} is the inertial dimension of the liquid; m_{4*} is inertial dimension (mass) of a liquid attached to a body of mass m_{2*}; m_v is the inertial dimension of the liquid displaced by the body surrounding the liquid; b_S, b_f are coefficients of viscosity; c_{1*} and c_{2*} are stiffnesses.

It is known that parametric excitations differ from those in which mechanical systems are directly affected by force, since the law of vibrations includes external influences as a modulation of the parameter [15, 16]. Given this, we write the expressions of the forces acting on the system under consideration as follows:

$$F_L=F_R=m_iW_0=-m_i\omega_{*m}^2\xi_0(t)q_i(t), \qquad (2)$$

where W_0 is the base acceleration; ω_{*m} are natural frequencies; $\xi_0(t)$ represents a stationary normal random process and is dimensionless.

Since $\xi_0(t)$ is stationary normal random process, its mathematical expectation is zero, i.e. $\langle\xi_0(t)\rangle=0$.

If we put the expression of forces (2) in the system of differential equations (1), the differential equations of motion of the rod protected from vibrations under the influence of random parametric excitations are as follows:

$$A\ddot{Q} + B\dot{Q} + C_{*}Q = 0, \tag{3}$$

where

$$C_{*} = \begin{bmatrix} c_{*i} + m_i\omega_{*m}^2\big(u_i(0) + u_i(L)\big)\xi_0(t) & -u_i(x_1)c_{1*} & 0 \\ 0 & c_{1*} & 0 \\ 0 & 0 & 2c_{2*} \end{bmatrix};$$

$c_{*i} = c_{1i} + Jc_{2i}$; c_{1i} and c_{2i} are the statistical linearization coefficients [17].

We express the variables in the system of equations (3) as follows:

$$\begin{gathered} q_{i*} = \sigma_i(t)e^{j\omega t} + \xi_i(t)e^{-j\omega t}; \\ q_3 = \sigma_{3a}(t)e^{j\omega t} + \xi_{3a}(t)e^{-j\omega t}; \\ q_4 = \sigma_{4a}(t)e^{j\omega t} + \xi_{4a}(t)e^{-j\omega t}, \end{gathered} \tag{4}$$

where ω is frequency; $\sigma_i(t), \xi_i(t), \sigma_{3a}(t), \xi_{3a}(t), \sigma_{4a}(t), \xi_{4a}(t)$ are slowly varying functions, and the amplitude value of the random parametric excitations of the rod is satisfies $\langle\zeta_{ia}\rangle = 2\sqrt{\langle\sigma_i(t)\rangle\langle\xi_i(t)\rangle}$ the condition [15].

Given that $\sigma_i(t), \xi_i(t), \sigma_{3a}(t), \xi_{3a}(t), \sigma_{4a}(t), \xi_{4a}(t)$ are slowly varying functions, the first-order derivatives from solutions (4) are as follows:

$$\begin{gathered} \dot{q}_{i*} = j\omega\big(\sigma_i(t)e^{j\omega t} - \xi_i(t)e^{-j\omega t}\big); \\ \dot{q}_3 = j\omega\big(\sigma_{3a}(t)e^{j\omega t} - \xi_{3a}(t)e^{-j\omega t}\big); \\ \dot{q}_4 = j\omega\big(\sigma_{4a}(t)e^{j\omega t} - \xi_{4a}(t)e^{-j\omega t}\big). \end{gathered} \tag{5}$$

If we put the variables (4) and their corresponding derivatives in the system of equations (3), we have the following first-order system of differential equations:

$$\begin{aligned} \dot{\sigma}_i = \frac{1}{2jm_i\omega}\Big(& (m_i\omega^2 - c_i)\big(\sigma_i + \xi_i e^{-2j\omega t}\big) + \big(c_1 + j\omega b_f\big)u_i(x_1)\sigma_{3a} \\ & + \big(c_1 - j\omega b_f\big)u_i(x_1)\xi_{3a}e^{-2j\omega t} \\ & + m_i\omega_{*m}^2\big(u_i(0) + u_i(L)\big)\xi_0(t)\big(\sigma_i + \xi_i e^{-2j\omega t}\big)\Big); \end{aligned}$$

$$\begin{aligned}\dot{\xi}_i = -\frac{1}{2jm_i\omega}\Big(&(m_i\omega^2 - c_i)\big(\sigma_i e^{2j\omega t} + \xi_i\big) + \big(c_1 + j\omega b_f\big)u_i(x_1)\sigma_{3a}e^{2j\omega t}\\ &+ \big(c_1 - j\omega b_f\big)u_i(x_1)\xi_{3a}\\ &+ m_i\omega_{*m}^2\big(u_i(0) + u_i(L)\big)\xi_0(t)\big(\sigma_i e^{2j\omega t} + \xi_i\big)\Big);\end{aligned}$$

$$\begin{aligned}\dot{\sigma}_{3a} = \frac{1}{2j\Delta\omega}\Bigg(&\frac{\Delta u_i(x_1)c_i}{m_i}\big(\sigma_i + \xi_i e^{-2j\omega t}\big)\\ &+ \left(\Delta\omega^2 - \big(c_1 + j\omega b_f\big)(M_4 + \frac{\Delta u_i^2(x_1)}{m_i})\right)\sigma_{3a}\\ &+ \left(\Delta\omega^2 - \big(c_1 - j\omega b_f\big)(M_4 + \frac{\Delta u_i^2(x_1)}{m_i})\right)\xi_{3a}e^{-2j\omega t}\\ &+ (2c_2 + j\omega b_s)M_2\sigma_{4a} + (2c_2 + j\omega b_s)M_2\xi_{4a}e^{-2j\omega t}\\ &- u_i(x_1)\omega_{*m}^2\Delta\big(u_i(0) + u_i(L)\big)\xi_0(t)\big(\sigma_i + \xi_i e^{-2j\omega t}\big)\Bigg);\end{aligned} \tag{6}$$

$$\begin{aligned}\dot{\xi}_{3a} = -\frac{1}{2j\Delta\omega}\Bigg(&\frac{\Delta u_i(x_1)c_i}{m_i}\big(\sigma_i e^{2j\omega t} + \xi_i\big)\\ &+ \left(\Delta\omega^2 - \big(c_1 + j\omega b_f\big)(M_4 + \frac{\Delta u_i^2(x_1)}{m_i})\right)\sigma_{3a}e^{2j\omega t}\\ &+ \left(\Delta\omega^2 - \big(c_1 - j\omega b_f\big)(M_4 + \frac{\Delta u_i^2(x_1)}{m_i})\right)\xi_{3a}\\ &+ (2c_2 + j\omega b_s)M_2\sigma_{4a}e^{2j\omega t} + (2c_2 + j\omega b_s)M_2\xi_{4a}\\ &- u_i(x_1)\omega_{*m}^2\Delta\big(u_i(0) + u_i(L)\big)\xi_0(t)\big(\sigma_i e^{2j\omega t} + \xi_i\big)\Bigg);\end{aligned}$$

$$\begin{aligned}\dot{\sigma}_{4a} = \frac{1}{2j\Delta\omega}\Big(&(\Delta\omega^2 - (2c_2 + j\omega b_s)M_1)\sigma_{4a}\\ &+ (\Delta\omega^2 - (2c_2 - j\omega b_s)M_1)\xi_{4a}e^{-2j\omega t} + \big(c_1 + j\omega b_f\big)M_3\sigma_{3a}\\ &+ \big(c_1 - j\omega b_f\big)M_3\xi_{3a}e^{-2j\omega t}\Big);\end{aligned}$$

$$\dot{\xi}_{4a} = -\frac{1}{2j\Delta\omega}\Big((\Delta\omega^2 - (2c_2 + j\omega b_s)M_1)\sigma_{4a}e^{2j\omega t} + (\Delta\omega^2 - (2c_2 - j\omega b_s)M_1)\xi_{4a} + \left(c_1 + j\omega b_f\right)M_3\sigma_{3a}e^{2j\omega t} + \left(c_1 - j\omega b_f\right)M_3\xi_{3a}\Big),$$

where $M_1 = m_{13} + m_{2*}; M_2 = m_{2*} + m_v;\ M_3 = m_{2*} - m_v;\ M_4 = m_{2*} + m_{4*};\ \Delta = M_1M_4 - M_2M_3$.

In stochastic processes, the variables of the system of equations (6) satisfy the Ito equations. Therefore, using the method of stochastic averaging, we write the system of equations (6) as the system of Ito equations. To do this, we write the system of equations (6) as follows:

$$\dot{X}_s(t) = f_s(X,t) + \sum_{r=1}^{6} G_{sr}(X,t)\xi_{0r}(t), \qquad (s = 1.,\dots,6) \tag{7}$$

where $X_1 = \sigma_i, X_2 = \xi_i, X_3 = \sigma_{3a}, X_4 = \xi_{3a}, X_5 = \sigma_{4a}, X_6 = \xi_{4a}$,

$$f_1(X,t) = l_1\sigma_i + l_2\sigma_{3a} + l_3\xi_i e^{-2j\omega t} + l_4\xi_{3a}e^{-2j\omega t};$$

$$f_2(X,t) = -l_1\sigma_i e^{2j\omega t} - l_2\sigma_{3a}e^{2j\omega t} - l_3\xi_i - l_4\xi_{3a};$$

$$f_3(X,t) = l_6\sigma_{3a} + l_7\sigma_{4a} + l_8\xi_{3a}e^{-2j\omega t} + l_9\xi_{4a}e^{-2j\omega t} + l_{10}\sigma_i + l_{11}\xi_i e^{-2j\omega t};$$

$$f_4(X,t) = -(l_6\sigma_{3a}e^{2j\omega t} + l_7\sigma_{4a}e^{2j\omega t} + l_8\xi_{3a} + l_9\xi_{4a} + l_{10}\sigma_i e^{2j\omega t} + l_{11}\xi_i);$$

$$f_5(X,t) = l_{13}\sigma_{4a} + l_{14}\xi_{4a}e^{-2j\omega t} + l_{15}\sigma_{3a} + l_{16}\xi_{3a}e^{-2j\omega t};$$

$$f_6(X,t) = -l_{13}\sigma_{4a}e^{2j\omega t} - l_{14}\xi_{4a} - l_{15}\sigma_{3a}e^{2j\omega t} - l_{16}\xi_{3a};$$

$$s \neq r, G_{sr}(X,t) = 0; G_{11}(X,t) = l_5\left(\sigma_i + \xi_i e^{-2j\omega t}\right);$$

$$G_{22}(X,t) = -l_5\left(\sigma_i e^{2j\omega t} + \xi_i\right); G_{33}(X,t) = u_i(x_1)l_{12}\left(\sigma_i + \xi_i e^{-2j\omega t}\right);$$

$$G_{44}(X,t) = -u_i(x_1)l_{12}\left(\sigma_i e^{2j\omega t} + \xi_i\right); G_{55}(X,t) = 0, G_{66}(X,t) = 0, \xi_{0r}(t) = \xi_0(t);$$

$$l_1 = l_3 = \frac{m_i\omega^2 - c_i}{2jm_i\omega}; l_2 = \frac{\left(c_1 + j\omega b_f\right)u_i(x_1)}{2jm_i\omega}; l_4 = \frac{\left(c_1 - j\omega b_f\right)u_i(x_1)}{2jm_i\omega};$$

$$l_5 = \frac{\omega_{*m}^2\left(u_i(0) + u_i(L)\right)}{2j\omega}; l_6 = \frac{\Delta\omega^2 - \left(c_1 + j\omega b_f\right)\left(M_4 + \frac{\Delta u_i^2(x_1)}{m_i}\right)}{2j\Delta\omega};$$

$$l_7 = \frac{(2c_2 + j\omega b_s)M_2}{2j\Delta\omega}; l_8 = \frac{\Delta\omega^2 - (c_1 - j\omega b_f)\left(M_4 + \frac{\Delta u_i^2(x_1)}{m_i}\right)}{2j\Delta\omega};$$

$$l_9 = \frac{(2c_2 - j\omega b_s)M_2}{2j\Delta\omega}; l_{10} = l_{11} = \frac{u_i(x_1)c_i}{2j\omega m_i};$$

$$l_{12} = -\frac{\omega_{*m}^2\left(u_i(0) + u_i(L)\right)}{2j\omega}; l_{13} = \frac{\Delta\omega^2 - (2c_2 + j\omega b_s)M_1}{2j\Delta\omega};$$

$$l_{14} = \frac{\Delta\omega^2 - (2c_2 - j\omega b_s)M_1}{2j\Delta\omega}; l_{15} = \frac{(c_1 + j\omega b_f)M_3}{2j\Delta\omega}; l_{16} = \frac{(c_1 - j\omega b_f)M_3}{2j\Delta\omega}.$$

We averaging equations (7) stochastically.

$$dX_s(t) = Y_s(X)dt + \sum_{r=1}^{6} H_{sr}(X)d\xi_{0r}(t), \qquad (s = 1., \dots, 6) \qquad (8)$$

where

$$Y_s = M_t\left\{f_s(X,t) + \sum_{l=1}^{6}\sum_{m,n=1}^{6}\int_{-\infty}^{0} G_{lm}(X, t+\tau)\frac{\partial G_{sn}(X,t)}{\partial X_l}E[\xi_{0n}(t)\xi_{0m}(t+\tau)]d\tau\right\}; \qquad (9)$$

$$[HH^T]_{sr} = M_t\left\{\sum_{m,n=1}^{6}\int_{-\infty}^{\infty} G_{sn}(X,t)G_{rm}(X,t+\tau)E[\xi_{0n}(t)\xi_{0m}(t+\tau)]d\tau\right\}; \quad (10)$$

$M_t\{\cdot\} = \lim_{n\to\infty}\frac{1}{T}\int_0^T\{\cdot\}dt$ is time averaging operator; $E[\cdot]$ is mathematical expectation; τ is the correlation time.

If we calculate the mathematical expectations of the variables in equations (8), since $\xi_{0r}(t) = \xi_0(t)$ is stationary normal random process, $\langle d\xi_{0r}(t)\rangle = d\langle\xi_{0r}(t)\rangle = d\langle\xi_0(t)\rangle = 0$ and the result we will have

$$\frac{d\langle X_s(t)\rangle}{dt} = Y_s(\langle X\rangle). \qquad (s = 1., \dots, 6) \qquad (11)$$

Namely,

$$\frac{d\langle\sigma_i\rangle}{dt} = l_1\langle\sigma_i\rangle + l_2\langle\sigma_{3a}\rangle + \frac{\pi l_{12}^2}{2}(S(0) - S(2\omega) - j\psi(2\omega))\langle\sigma_i\rangle;$$

$$\frac{d\langle\xi_i\rangle}{dt} = -l_3\langle\xi_i\rangle - l_4\langle\xi_{3a}\rangle + \frac{\pi l_{12}^2}{2}(S(0) - S(2\omega) + j\psi(2\omega))\langle\xi_i\rangle;$$

$$\frac{d\langle\sigma_{3a}\rangle}{dt} = l_6\langle\sigma_{3a}\rangle + l_7\langle\sigma_{4a}\rangle + l_{10}\langle\sigma_i\rangle - \frac{\pi u_i(x_1)l_{12}^2}{2}\big(S(0) - S(2\omega) - j\psi(2\omega)\big)\langle\sigma_i\rangle; \tag{12}$$

$$\frac{d\langle\xi_{3a}\rangle}{dt} = -l_8\langle\xi_{3a}\rangle - l_9\langle\xi_{4a}\rangle - l_{11}\langle\sigma_i\rangle - \frac{\pi u_i(x_1)l_{12}^2}{2}(S(0) - S(2\omega) + j\psi(2\omega))\langle\xi_i\rangle;$$

$$\frac{d\langle\sigma_{4a}\rangle}{dt} = l_{13}\langle\sigma_{4a}\rangle + l_{15}\langle\sigma_{3a}\rangle;$$

$$\frac{d\langle\xi_{4a}\rangle}{dt} = -l_{14}\langle\xi_{4a}\rangle - l_{16}\langle\xi_{3a}\rangle,$$

where $S(0), S(2\omega), \psi(2\omega)$ are the spectral densities of stationary normal random process $\xi_0(t)$ defined as follows [18]:

$$S(2\omega) = \frac{1}{\pi}\int_{-\infty}^{0} R(\tau)cos\omega\tau d\tau;$$

$$\psi(2\omega) = \frac{1}{\pi}\int_{-\infty}^{0} R(\tau)sin\omega\tau d\tau;$$

$R(\tau) = E[\xi_{0n}(t)\xi_{0m}(t+\tau)] = \langle\xi_{0n}(t)\xi_{0m}(t+\tau)\rangle$ is a correlation function.

We look for the solution of the system of equations (12) as follows:

$$\begin{gathered}\sigma_i(t) = \sigma_{i0}e^{\lambda t}; \xi_i(t) = \xi_{i0}e^{\lambda t};\\ \sigma_{3a}(t) = \sigma_{3*}e^{\lambda t}; \xi_{3a}(t) = \xi_{3*}e^{\lambda t};\\ \sigma_{4a}(t) = \sigma_{4*}e^{\lambda t}; \xi_{4a}(t) = \xi_{4*}e^{\lambda t},\end{gathered} \tag{13}$$

We put the solutions (13) in the system of equations (12) and the determinant view of the characteristic equation of the system can be formed as follows, provided that we have a non-zero solution:

$$\begin{vmatrix} b_{11}-\lambda & \cdots & b_{16} \\ \vdots & \ddots & \vdots \\ b_{61} & \cdots & b_{66}-\lambda \end{vmatrix} = 0, \tag{14}$$

where

$$b_{11} = l_1 + \frac{\pi l_{12}^2}{2}\big(S(0) - S(2\omega) - j\psi(2\omega)\big); b_{12} = 0;$$

$$b_{13} = l_2; b_{14} = b_{15} = b_{16} = 0;$$

$$b_{21} = 0; b_{22} = -l_3 + \frac{\pi l_{12}^2}{2}\big(S(0) - S(2\omega) + j\psi(2\omega)\big);$$

$$b_{23} = 0; b_{24} = -l_4; b_{25} = b_{26} = 0;$$

$$b_{31} = l_{10} - \frac{\pi u_i(x_1) l_{12}^2}{2}\big(S(0) - S(2\omega) - j\psi(2\omega)\big);$$

$$b_{32} = 0; b_{33} = l_6; b_{34} = 0; b_{35} = l_7; b_{36} = 0;$$

$$b_{41} = 0; b_{42} = -l_{11} - \frac{\pi u_i(x_1) l_{12}^2}{2}\big(S(0) - S(2\omega) + j\psi(2\omega)\big);$$

$$b_{43} = 0; b_{44} = -l_8; b_{45} = 0; b_{46} = -l_9;$$

$$b_{51} = b_{52} = 0; b_{53} = l_{15}; b_{54} = 0; b_{55} = l_{13}; b_{56} = 0;$$

$$b_{61} = b_{62} = b_{63} = 0; b_{64} = -l_{16}; b_{65} = 0; b_{66} = -l_{14}.$$

We find the value of this determinant by zeroing all the elements on one side of the determinant (14) diagonal. As a result, its value is equal to the product of the diagonal elements, resulting in the following two characteristic equations:

$$\lambda^3 + y_i\lambda^2 + n_i\lambda + k_i = 0, (i = 1{,}2). \qquad (15)$$

where $y_1 = -(b_{11} + b_{33} + b_{55})$; $n_1 = -(b_{35}b_{53} + b_{13}b_{31} - b_{11}b_{33} - b_{11}b_{55} - b_{33}b_{55})$; $k_1 = -(b_{11}b_{33}b_{55} - b_{55}b_{31}b_{13} - b_{35}b_{53}b_{11})$; $y_2 = -(b_{22} + b_{44} + b_{66})$; $n_2 = b_{22}b_{44} - b_{42}b_{24} + b_{66}b_{22} + b_{66}b_{44} - b_{46}b_{64}$; $k_2 = -(b_{22}b_{44}b_{66} - b_{66}b_{42}b_{24} - b_{46}b_{64}b_{52})$.

The obtained characteristic equations allow to explore the stability and dynamics of the system under consideration.

Results and discussion. Characteristic equations are cubic equations. We identify their roots. As a result, the roots of the characteristic equations (15) are as follows:

$$\lambda_1 = -\frac{y_1}{3} + A_{1*} + B_{1*};$$

$$\lambda_{2,3} = -\frac{y_1}{3} - \frac{A_{1*} + B_{1*}}{2} \pm \sqrt{-\frac{3(A_{1*} - B_{1*})}{2}};$$

$$\lambda_4 = -\frac{y_2}{3} + A_{2*} + B_{2*}; \qquad (16)$$

$$\lambda_{5,6}=-\frac{y_2}{3}-\frac{A_{2*}+B_{2*}}{2}\pm\sqrt{-\frac{3(A_{2*}-B_{2*})}{2}};$$

$$A_{1*}=(-\frac{p_1}{2}+(\frac{p_1^2}{4}+\frac{p_*^3}{27}))^{1/3};B_{1*}=(-\frac{p_1}{2}-(\frac{p_1^2}{4}+\frac{p_*^3}{27}))^{1/3};$$

$$p_1=\frac{2y_1^3-9y_1n_1+27k_1}{27};p_*=\frac{3n_1-y_1^2}{3};$$

$$A_{2*}=(-\frac{p_2}{2}+(\frac{p_2^2}{4}+\frac{p_{**}^3}{27}))^{1/3};B_{2*}=(-\frac{p_2}{2}-(\frac{p_2^2}{4}+\frac{p_{**}^3}{27}))^{1/3};$$

$$p_2=\frac{2y_2^3-9y_2n_2+27k_2}{27};p_{**}=\frac{3n_2-y_2^2}{3}.$$

We distinguish between real and abstract parts of the roots of the characteristic equation. Initially, for this, the functions p_1, p_*, p_2, p_{**} can be expressed in complex form.

$$p_1=2(R_1+jL_1);p_*=3(R_2+jL_2);p_2=2(R_3+jL_3);p_{**}=3(R_4+jL_4).$$

According to that

$$A_{1*}=\left(-3L_2^2R_2+R_2^3-L_1^2+R_1^2-R_1+J(3R_2^2L_2-L_2^3+2L_1R_1-L_1)\right)^{\frac{1}{3}};$$

$$B_{1*}=\left(3L_2^2R_2-R_2^3+L_1^2-R_1^2-R_1+J(-3R_2^2L_2+L_2^3-2L_1R_1-L_1)\right)^{\frac{1}{3}};$$

$$A_{2*}=\left(-3L_4^2R_4+R_4^3-L_3^2+R_3^2-R_3+J(3R_4^2L_4-L_4^3+2L_3R_3-L_3)\right)^{\frac{1}{3}};$$

$$B_{2*}=\left(3L_4^2R_4-R_4^3+L_3^2-R_3^2-R_3+J(-3R_4^2L_4+L_4^3-2L_3R_3-L_3)\right)^{\frac{1}{3}}.$$

We determine the roots of the resulting complex expressions using the De Moivre's formula.

$$A_{1*}=((-3L_2^2R_2+R_2^3-L_1^2+R_1^2-R_1)^2 \\ +(3R_2^2L_2-L_2^3+2L_1R_1-L_1)^2)^{\frac{1}{6}}(cos\frac{\varphi_1}{3}+jsin\frac{\varphi_1}{3});$$

$$B_{1*}=((3L_2^2R_2-R_2^3+L_1^2-R_1^2-R_1)^2 \\ +(-3R_2^2L_2+L_2^3-2L_1R_1-L_1)^2)^{\frac{1}{6}}(cos\frac{\varphi_2}{3}+jsin\frac{\varphi_2}{3});$$

$$\begin{aligned} A_{2*} = ((-3L_4^2R_4 + R_4^3 - L_3^2 + R_3^2 - R_3)^2 \\ + (3R_4^2L_4 - L_4^3 + 2L_3R_3 - L_3)^2)^{\frac{1}{6}}(cos\frac{\varphi_3}{3} + jsin\frac{\varphi_3}{3}); \end{aligned}$$

$$\begin{aligned} B_{2*} = ((3L_4^2R_4 - R_4^3 + L_3^2 - R_3^2 - R_3)^2 \\ + (-3R_4^2L_4 + L_4^3 - 2L_3R_3 - L_3)^2)^{\frac{1}{6}}\left(cos\frac{\varphi_4}{3} + jsin\frac{\varphi_4}{3}\right), \end{aligned}$$

where

$$cos\varphi_1 = \frac{-3L_2^2R_2 + R_2^3 - L_1^2 + R_1^2 - R_1}{((-3L_2^2R_2 + R_2^3 - L_1^2 + R_1^2 - R_1)^2 + (3R_2^2L_2 - L_2^3 + 2L_1R_1 - L_1)^2)^{\frac{1}{2}}};$$

$$sin\varphi_1 = \frac{3R_2^2L_2 - L_2^3 + 2L_1R_1 - L_1}{((-3L_2^2R_2 + R_2^3 - L_1^2 + R_1^2 - R_1)^2 + (3R_2^2L_2 - L_2^3 + 2L_1R_1 - L_1)^2)^{\frac{1}{2}}};$$

$$cos\varphi_2 = \frac{3L_2^2R_2 - R_2^3 + L_1^2 - R_1^2 - R_1}{((3L_2^2R_2 - R_2^3 + L_1^2 - R_1^2 - R_1)^2 + (-3R_2^2L_2 + L_2^3 - 2L_1R_1 - L_1)^2)^{\frac{1}{2}}};$$

$$sin\varphi_2 = \frac{-3R_2^2L_2 + L_2^3 - 2L_1R_1 - L_1}{((3L_2^2R_2 - R_2^3 + L_1^2 - R_1^2 - R_1)^2 + (-3R_2^2L_2 + L_2^3 - 2L_1R_1 - L_1)^2)^{\frac{1}{2}}};$$

$$cos\varphi_3 = \frac{-3L_4^2R_4 + R_4^3 - L_3^2 + R_3^2 - R_3}{((-3L_4^2R_4 + R_4^3 - L_3^2 + R_3^2 - R_3)^2 + (3R_4^2L_4 - L_4^3 + 2L_3R_3 - L_3)^2)^{\frac{1}{2}}};$$

$$sin\varphi_3 = \frac{3R_4^2L_4 - L_4^3 + 2L_3R_3 - L_3}{((-3L_4^2R_4 + R_4^3 - L_3^2 + R_3^2 - R_3)^2 + (3R_4^2L_4 - L_4^3 + 2L_3R_3 - L_3)^2)^{\frac{1}{2}}};$$

$$cos\varphi_4 = \frac{3L_4^2R_4 - R_4^3 + L_3^2 - R_3^2 - R_3}{((3L_4^2R_4 - R_4^3 + L_3^2 - R_3^2 - R_3)^2 + (-3R_4^2L_4 + L_4^3 - 2L_3R_3 - L_3)^2)^{\frac{1}{2}}};$$

$$sin\varphi_4 = \frac{-3R_4^2L_4 + L_4^3 - 2L_3R_3 - L_3}{((3L_4^2R_4 - R_4^3 + L_3^2 - R_3^2 - R_3)^2 + (-3R_4^2L_4 + L_4^3 - 2L_3R_3 - L_3)^2)^{\frac{1}{2}}}.$$

In that case

$$A_{1*} + B_{1*} = A_{10}cos\frac{\varphi_1}{3} + B_{10}cos\frac{\varphi_2}{3} + j\left(A_{10}sin\frac{\varphi_1}{3} + B_{10}sin\frac{\varphi_2}{3}\right);$$

$$A_{2*} + B_{2*} = A_{20}cos\frac{\varphi_3}{3} + B_{20}cos\frac{\varphi_4}{3} + j\left(A_{20}sin\frac{\varphi_3}{3} + B_{20}sin\frac{\varphi_4}{3}\right); \qquad (17)$$

$$\sqrt{\frac{-3(A_{1*}-B_{1*})}{2}}=\sqrt{\frac{-3(A_{10}cos\frac{\varphi_1}{3}-B_{10}cos\frac{\varphi_2}{3}+j\left(A_{10}sin\frac{\varphi_1}{3}-B_{10}sin\frac{\varphi_2}{3}\right))}{2}}$$

$$=C_{10}(Jcos\frac{\varphi_5}{2}-sin\frac{\varphi_5}{2});$$

$$\sqrt{\frac{-3(A_{2*}-B_{2*})}{2}}=\sqrt{\frac{3(A_{20}cos\frac{\varphi_3}{3}-B_{20}cos\frac{\varphi_4}{3}+j\left(A_{20}sin\frac{\varphi_3}{3}-B_{20}sin\frac{\varphi_4}{3}\right))}{-2}}$$

$$=C_{20}(Jcos\frac{\varphi_6}{2}-sin\frac{\varphi_6}{2});$$

$$cos\varphi_5=\frac{A_{10}cos\frac{\varphi_1}{3}-B_{10}cos\frac{\varphi_2}{3}}{\left(A_{10}^2+B_{10}^2-2A_{10}B_{10}\cos\frac{(\varphi_1-\varphi_2)}{3}\right)^{\frac{1}{2}}};$$

$$sin\varphi_5=\frac{A_{10}sin\frac{\varphi_1}{3}-B_{10}sin\frac{\varphi_2}{3}}{\left(A_{10}^2+B_{10}^2-2A_{10}B_{10}\cos\frac{(\varphi_1-\varphi_2)}{3}\right)^{\frac{1}{2}}};$$

$$cos\varphi_6=\frac{A_{20}cos\frac{\varphi_3}{3}-B_{20}cos\frac{\varphi_4}{3}}{\left(A_{20}^2+B_{20}^2-2A_{20}B_{20}\cos\frac{(\varphi_3-\varphi_4)}{3}\right)^{\frac{1}{2}}};$$

$$sin\varphi_6=\frac{A_{20}sin\frac{\varphi_3}{3}-B_{20}sin\frac{\varphi_4}{3}}{\left(A_{20}^2+B_{20}^2-2A_{20}B_{20}\cos\frac{(\varphi_3-\varphi_4)}{3}\right)^{\frac{1}{2}}};$$

$$A_{10}=((-3L_2^2R_2+R_2^3-L_1^2+R_1^2-R_1)^2+(3R_2^2L_2-L_2^3+2L_1R_1-L_1)^2)^{\frac{1}{6}};$$

$$B_{10}=((3L_2^2R_2-R_2^3+L_1^2-R_1^2-R_1)^2+(-3R_2^2L_2+L_2^3-2L_1R_1-L_1)^2)^{\frac{1}{6}};$$

$$A_{20}=((-3L_4^2R_4+R_4^3-L_3^2+R_3^2-R_3)^2+(3R_4^2L_4-L_4^3+2L_3R_3-L_3)^2)^{\frac{1}{6}};$$

$$B_{20}=((3L_4^2R_4-R_4^3+L_3^2-R_3^2-R_3)^2+(-3R_4^2L_4+L_4^3-2L_3R_3-L_3)^2)^{\frac{1}{6}}.$$

$$C_{10}=\sqrt{\frac{3}{2}}\left(A_{10}^2+B_{10}^2-2A_{10}B_{10}\cos\frac{(\varphi_1-\varphi_2)}{3}\right)^{\frac{1}{4}};$$

$$C_{20}=\sqrt{\frac{3}{2}}\left(A_{20}^2+B_{20}^2-2A_{20}B_{20}\cos\frac{(\varphi_3-\varphi_4)}{3}\right)^{\frac{1}{4}}.$$

We put expressions (17) into expressions of roots (16).

$$\lambda_1=-\frac{y_1}{3}+A_{10}cos\frac{\varphi_1}{3}+B_{10}cos\frac{\varphi_2}{3}+j\left(A_{10}sin\frac{\varphi_1}{3}+B_{10}sin\frac{\varphi_2}{3}\right);$$

$$\lambda_{2,3}=-\frac{y_1}{3}-\frac{(A_{10}cos\frac{\varphi_1}{3}+B_{10}cos\frac{\varphi_2}{3}+j\left(A_{10}sin\frac{\varphi_1}{3}+B_{10}sin\frac{\varphi_2}{3}\right))}{2}$$

$$\pm C_{10}(Jcos\frac{\varphi_5}{2}-sin\frac{\varphi_5}{2});$$

$$\lambda_4=-\frac{y_2}{3}+A_{20}cos\frac{\varphi_3}{3}+B_{20}cos\frac{\varphi_4}{3}+j\left(A_{20}sin\frac{\varphi_3}{3}+B_{20}sin\frac{\varphi_4}{3}\right);\qquad(18)$$

$$\lambda_{5,6}=-\frac{y_2}{3}-\frac{(A_{20}cos\frac{\varphi_3}{3}+B_{20}cos\frac{\varphi_4}{3}+j\left(A_{20}sin\frac{\varphi_3}{3}+B_{20}sin\frac{\varphi_4}{3}\right))}{2}$$

$$\pm C_{20}\left(Jcos\frac{\varphi_6}{2}-sin\frac{\varphi_6}{2}\right).$$

Taking into account the values of y_1 and y_2, the stability conditions of the system under consideration on the basis of the characteristic equation (18) are as follows:

$$\frac{1}{6}\left(\frac{c_{2i}}{m_i\omega}+\pi l_{12}^2\left(S(0)-S(2\omega)\right)-\frac{b_f\left(M_4+\frac{\Delta u_i^2(x_1)}{m_i}\right)}{\Delta}-\frac{b_sM_1}{\Delta}\right)+A_{10}cos\frac{\varphi_1}{3}$$

$$+B_{10}cos\frac{\varphi_2}{3}<0;$$

$$\frac{1}{6}\left(\frac{c_{2i}}{m_i\omega}+\pi l_{12}^2\left(S(0)-S(2\omega)\right)-\frac{b_f\left(M_4+\frac{\Delta u_i^2(x_1)}{m_i}\right)}{\Delta}-\frac{b_sM_1}{\Delta}\right)$$

$$-\frac{(A_{10}cos\frac{\varphi_1}{3}+B_{10}cos\frac{\varphi_2}{3})}{2}\pm C_{10}(-sin\frac{\varphi_5}{2})<0;$$

(19)

$$\frac{1}{6}\left(\frac{c_{2i}}{m_i\omega}+\pi l_{12}^2\big(S(0)-S(2\omega)\big)-\frac{b_f\left(M_4+\frac{\Delta u_i^2(x_1)}{m_i}\right)}{\Delta}-\frac{b_sM_1}{\Delta}\right)+A_{20}cos\frac{\varphi_3}{3}+B_{10}cos\frac{\varphi_4}{3}<0;$$

$$\frac{1}{6}\left(\frac{c_{2i}}{m_i\omega}+\pi l_{12}^2\big(S(0)-S(2\omega)\big)-\frac{b_f\left(M_4+\frac{\Delta u_i^2(x_1)}{m_i}\right)}{\Delta}-\frac{b_sM_1}{\Delta}\right)-\frac{(A_{20}cos\frac{\varphi_3}{3}+B_{20}cos\frac{\varphi_4}{3})}{2}\pm C_{20}\left(-sin\frac{\varphi_6}{2}\right)<0.$$

The following two inequalities can be written where the conditions of stability (19) are reasonable:

$$\begin{gathered}\frac{1}{6}\left(\frac{c_{2i}}{m_i\omega}+\pi l_{12}^2\big(S(0)-S(2\omega)\big)-\frac{b_f\left(M_4+\frac{\Delta u_i^2(x_1)}{m_i}\right)+b_sM_1}{\Delta}\right)-\frac{A_{10}cos\frac{\varphi_1}{3}+B_{10}cos\frac{\varphi_2}{3}}{2}<-C_{10}\left|sin\frac{\varphi_5}{2}\right|;\\ \frac{1}{6}\left(\frac{c_{2i}}{m_i\omega}+\pi l_{12}^2\big(S(0)-S(2\omega)\big)-\frac{b_f\left(M_4+\frac{\Delta u_i^2(x_1)}{m_i}\right)+b_sM_1}{\Delta}\right)-\frac{A_{20}cos\frac{\varphi_3}{3}+B_{20}cos\frac{\varphi_4}{3}}{2}<-C_{20}\left|sin\frac{\varphi_6}{2}\right|.\end{gathered}\tag{20}$$

The conditions obtained (20) are the stability condition of rod with an elastic dissipative characteristic of the hysteresis type, which is protected from vibrations under the influence of random parametric excitations.

Suppose

$$\frac{A_{20}cos\frac{\varphi_3}{3}+B_{20}cos\frac{\varphi_4}{3}}{2}-C_{20}\left|sin\frac{\varphi_6}{2}\right|>\frac{A_{10}cos\frac{\varphi_1}{3}+B_{10}cos\frac{\varphi_2}{3}}{2}-C_{10}\left|sin\frac{\varphi_5}{2}\right|,$$

let the attitude be reasonable. In this case, the first condition of the system of inequalities (20) represents the stability condition for the system under consideration.

The condition of instability for this case is as follows:

$$\frac{1}{6}\left(\frac{c_{2i}}{m_i\omega}+\pi l_{12}^2\big(S(0)-S(2\omega)\big)-\frac{b_f\left(M_4+\frac{\Delta u_i^2(x_1)}{m_i}\right)+b_sM_1}{\Delta}\right)$$

$$-\frac{A_{10}cos\frac{\varphi_1}{3}+B_{10}cos\frac{\varphi_2}{3}}{2}>-C_{10}\left|sin\frac{\varphi_5}{2}\right|. \qquad (21)$$

If

$$\frac{A_{20}cos\frac{\varphi_3}{3}+B_{20}cos\frac{\varphi_4}{3}}{2}-C_{20}\left|sin\frac{\varphi_6}{2}\right|<\frac{A_{10}cos\frac{\varphi_1}{3}+B_{10}cos\frac{\varphi_2}{3}}{2}-C_{10}\left|sin\frac{\varphi_5}{2}\right|,$$

the condition is reasonable, the second condition of the system of inequalities (20) represents the stability condition for the system under consideration. In this case, the condition of instability is as follows:

$$\frac{1}{6}\left(\frac{c_{2i}}{m_i\omega}+\pi l_{12}^2\big(S(0)-S(2\omega)\big)-\frac{b_f\left(M_4+\frac{\Delta u_i^2(x_1)}{m_i}\right)+b_sM_1}{\Delta}\right)$$

$$-\frac{A_{20}cos\frac{\varphi_3}{3}+B_{20}cos\frac{\varphi_4}{3}}{2}>-C_{20}\left|sin\frac{\varphi_6}{2}\right|. \qquad (22)$$

The function c_{1i} represents the vertical deviation of the amplitude-frequency characteristic, and the function c_{2i} represents the energy dissipation in the rod material [17]. In inequalities (21) and (22), when the argument $\langle\zeta_{ia}\rangle$ of the functions c_{1i} and c_{2i} reaches a value of $\langle\zeta_{ia*}\rangle$, stable vibrations are formed. For these values $\langle\zeta_{ia*}\rangle$ the following relation is appropriate:

$$\frac{1}{6}\left(\frac{c_{2i}}{m_i\omega}+\pi l_{12}^2\big(S(0)-S(2\omega)\big)-\frac{b_f\left(M_4+\frac{\Delta u_i^2(x_1)}{m_i}\right)+b_sM_1}{\Delta}\right)$$

$$-\frac{A_{10}cos\frac{\varphi_1}{3}+B_{10}cos\frac{\varphi_2}{3}}{2}=-C_{10}\left|sin\frac{\varphi_5}{2}\right|. \tag{23}$$

$$\frac{1}{6}\left(\frac{c_{2i}}{m_i\omega}+\pi l_{12}^2\big(S(0)-S(2\omega)\big)-\frac{b_f\left(M_4+\frac{\Delta u_i^2(x_1)}{m_i}\right)+b_sM_1}{\Delta}\right)$$

$$-\frac{A_{20}cos\frac{\varphi_3}{3}+B_{20}cos\frac{\varphi_4}{3}}{2}=-C_{20}\left|sin\frac{\varphi_6}{2}\right|. \tag{24}$$

Equations (23) and (24) allow us to determine the mean quadratic values of rod vibrations with elastic dissipative characteristics of the hysteresis type, protected from vibrations under the influence of random parametric excitations, and to fully analyze the dynamic nature.

Conclusion.

The stability conditions of rod vibrations with elastic dissipative characteristics of the hysteresis type, protected from vibrations under the influence of random parametric excitations, were determined depending on the system parameters. Stability borders were expressed analytically, which showed that they allow to determine the average quadratic values and check the dynamics of rod vibrations with elastic dissipative characteristics of the hysteresis type, protected from vibrations.

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